Theory of Structures

DWE-3xxx

Zaid Al-Azzawi, PhD

جامعة الانبار

1987 1408

UNIVERSITY OF ANBAR

COLLEGE OF ENGINEERING

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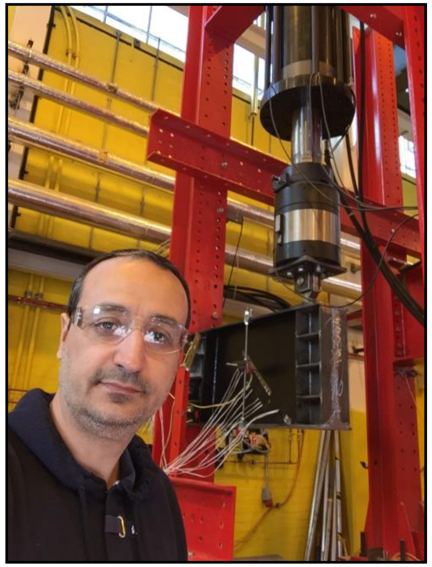
شركة توزيع الكهرباء م.ع.م
المبنى رقم (١)



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Course Description:

This course covers the outlines of general principles, indeterminacy and stability, shear and moment diagrams of structures, trusses, approximate analysis, influence lines and moving concentrated loads, analysis of statically determinate structures, analysis of statically indeterminate structures.

Course Objectives:

1. To impart the principles of elastic structural analysis and behaviour of indeterminate structures.
2. Ability to idealize and analyze statically determinate and indeterminate structures.
3. To enable the student to get a feeling of how real-life structures behave.
4. Familiarity with professional and contemporary issues.

Student Outcomes:

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The student after undergoing this course will be able to:

1. To understand analysis of indeterminate structures and adopt an appropriate structural analysis technique.

2. Determine response of structures by classical, iterative and matrix methods.

Text Book:

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Structural Analysis by R. C. Hibbeler- 8^{th} edition.

REFERENCES:

- Theory of Structures by S.P. Timoshenko and D. H. Young - 2^{nd} edition.
- Theory of Structures by Yuang Yu Hsiegh.
- Structural Analysis by Aslam Kassimali, 4^{th} edition.
- Structural and Stress Analysis by Dr. T.H.G Megson – 2^{nd} edition, 2000.

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Course Assessement:

Term Tests	Laboratory	Quizzes	Project	Final Exam
30.0%	0.0%	10.0%	----	60.0%

Syllabus

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week	Topics Covered
1	Introduction to structural analysis
2	Determinacy and stability of structures
3	Shear and moment diagrams of structures
4	Shear and moment diagrams of structures
5	Simple Trusses and Compound Trusses
6	Complex Trusses OR Approximate Analysis of Structures
7	Influence lines and moving concentrated loads
8	Influence lines and moving concentrated loads
9	Deflection of determinate structures
10	Deflection of determinate structures
11	Analysis of indeterminate structures- Consistent deformation method.
12	Analysis of indeterminate structures- Consistent deformation method.
13	Analysis of indeterminate structures using Slope-Deflection Method
14	Analysis of indeterminate structures using Moment-Distribution Method
15	Review

Unit-1

Introduction to Structural Analysis

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1.1 Types of Structural Forms

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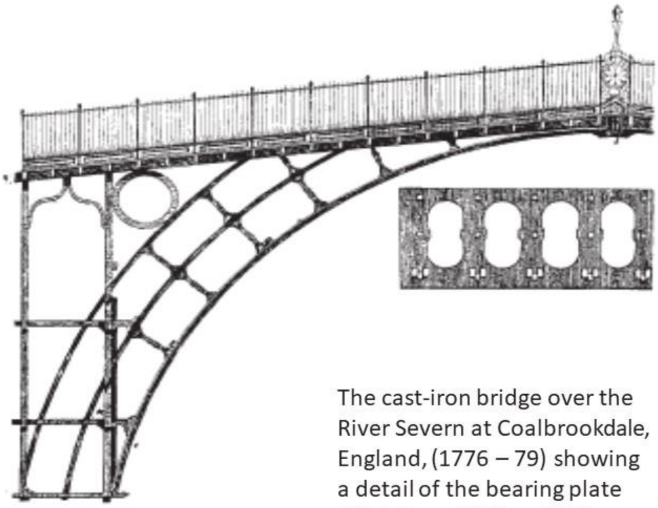

Suspension bridge over the Menai Strait near Bangor, Wales [Dietrich, 1998, p. 115]

The cast-iron bridge over the River Severn at Coalbrookdale, England, (1776 – 79) showing a detail of the bearing plate [Mehrtens, 1908, p. 270]

Röbling's Niagara Bridge [Güntheroth & Kahlow, 2005, p. 135]

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The first home of the Institute of Engineers of Ways of Communication and the Russian Highways Authority – Jusupov Palace on the River Fontanka, St. Petersburg [Fedorov, 2005, p. 57]

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Göltzsch Viaduct around 1850
[Conrad & Hänseroth, 1995, p. 762]

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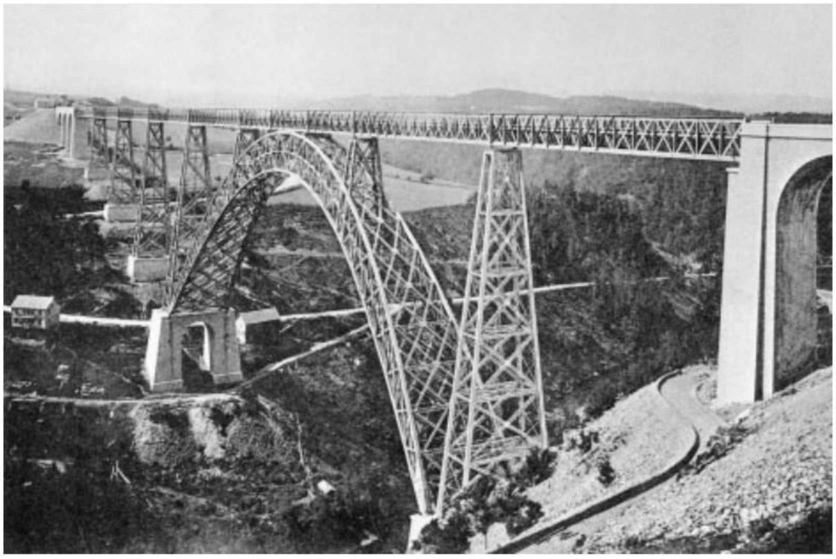

The Garabit Viaduct shortly after
completion [Eiffel, 1889]

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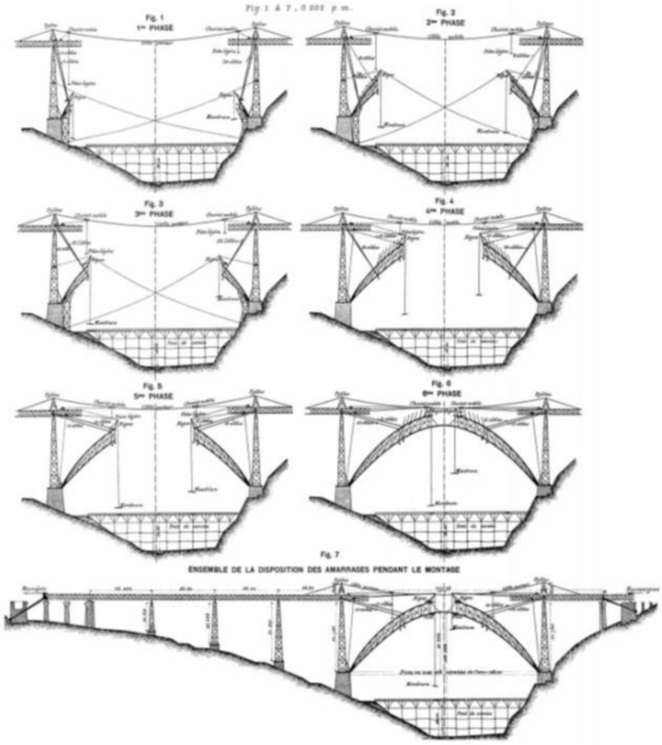
Fig. 1
1re PHASE
Fig. 2
2me PHASE
Fig. 3
3me PHASE
Fig. 4
4me PHASE
Fig. 5
5me PHASE
Fig. 6
6me PHASE
Fig. 7
ENSEMBLE DE LA DISPOSITION DES AMARRAGES PENDANT LE MONTAGE

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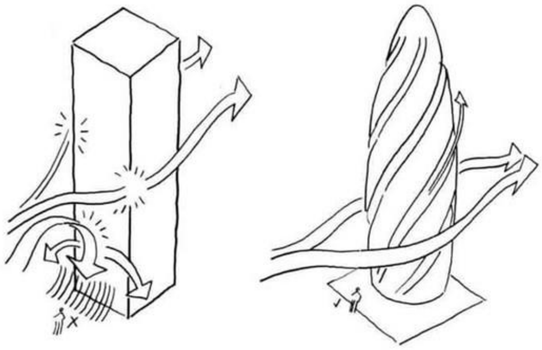

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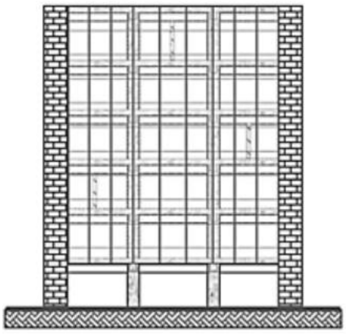
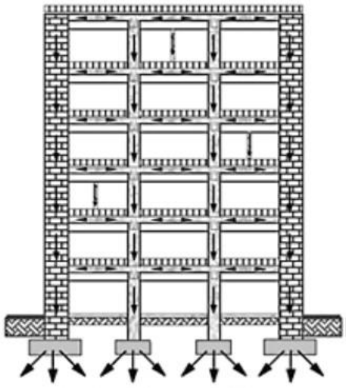
1.2 Loads





Load path for a typical frame

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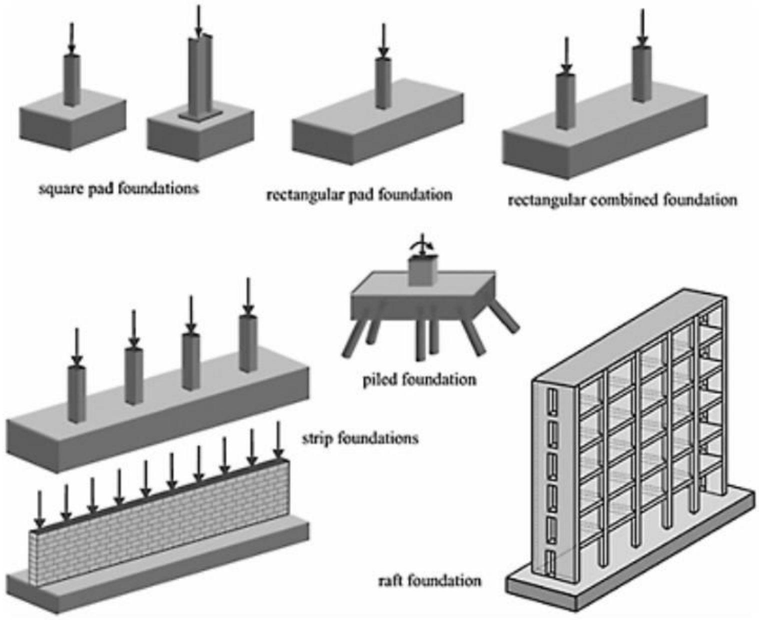

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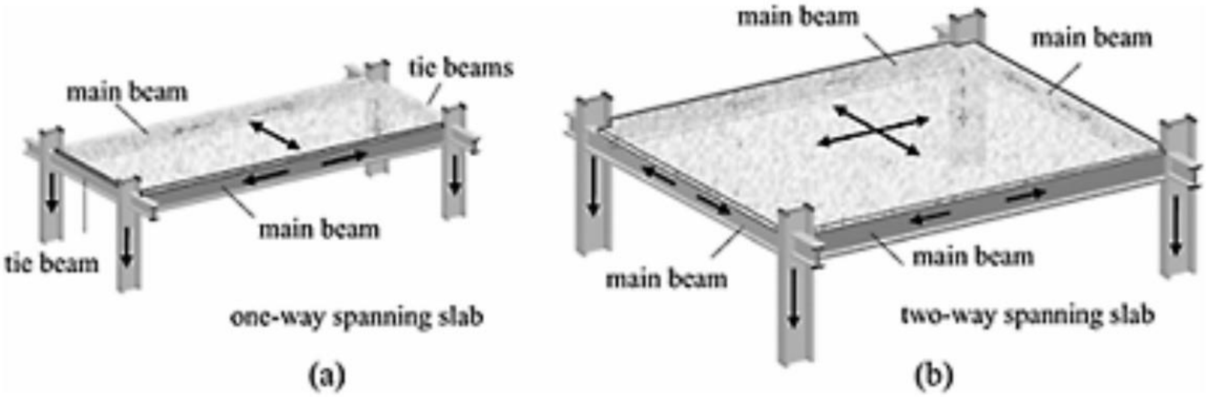
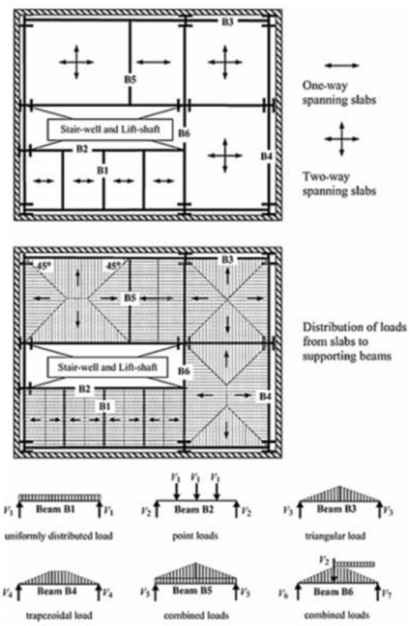


main beam
tie beams
main beam
tie beam
one-way spanning slab
(a)
main beam
main beam
main beam
main beam
two-way spanning slab
(b)



B3
B5
One-way spanning slabs
Stair-well and Lift-shaft
B6
B2
B1
B4
Two-way spanning slabs
45°
45°
Distribution of loads from slabs to supporting beams
Beam B1
uniformly distributed load
Beam B2
point loads
Beam B3
triangular load
Beam B4
trapezoidal load
Beam B5
combined loads
Beam B6
combined loads

- Dead Loads
- Live loads
- Moving loads
- Impact loads
- Wind loads
- Snaw loads
- Earthquake loads
- Blast loads
- Temperature load
- Soil pressure
- Hydrostatic load
- Centrifugal forces

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TABLE 1–1 Codes

General Building Codes
Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-10, American Society of Civil Engineers
International Building Code
Design Codes
Building Code Requirements for Reinforced Concrete, Am. Conc. Inst. (ACI)
Manual of Steel Construction, American Institute of Steel Construction (AISC)
Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials (AASHTO)
National Design Specification for Wood Construction, American Forest and Paper Association (AFPA)
Manual for Railway Engineering, American Railway Engineering Association (AREA)

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TABLE 1–2 Minimum Densities for Design Loads from Materials*

	lb/ft^3	kN/m^3
Aluminum	170	26.7
Concrete, plain cinder	108	17.0
Concrete, plain stone	144	22.6
Concrete, reinforced cinder	111	17.4
Concrete, reinforced stone	150	23.6
Clay, dry	63	9.9
Clay, damp	110	17.3
Sand and gravel, dry, loose	100	15.7
Sand and gravel, wet	120	18.9
Masonry, lightweight solid concrete	105	16.5
Masonry, normal weight	135	21.2
Plywood	36	5.7
Steel, cold-drawn	492	77.3
Wood, Douglas Fir	34	5.3
Wood, Southern Pine	37	5.8
Wood, spruce	29	4.5

*Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures,* ASCE/SEI 7-10. Copies of this standard may be purchased from ASCE at www.pubs.asce.org.

TABLE 1–3 Minimum Design Dead Loads*

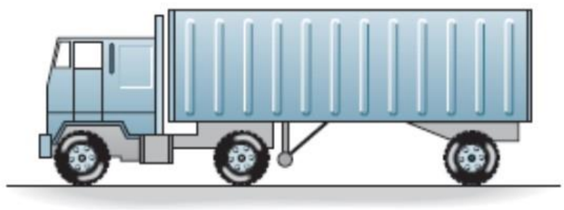
Walls	psf	kN/m^2
4-in. (102 mm) clay brick	39	1.87
8-in. (203 mm) clay brick	79	3.78
12-in. (305 mm) clay brick	115	5.51
Frame Partitions and Walls		
Exterior stud walls with brick veneer	48	2.30
Windows, glass, frame and sash	8	0.38
Wood studs 2 × 4 in., (51 × 102 mm) unplastered	4	0.19
Wood studs 2 × 4 in., (51 × 102 mm) plastered one side	12	0.57
Wood studs 2 × 4 in., (51 × 102 mm) plastered two sides	20	0.96
Floor Fill		
Cinder concrete, per inch (mm)	9	0.017
Lightweight concrete, plain, per inch (mm)	8	0.015
Stone concrete, per inch (mm)	12	0.023
Ceilings		
Acoustical fiberboard	1	0.05
Plaster on tile or concrete	5	0.24
Suspended metal lath and gypsum plaster	10	0.48
Asphalt shingles	2	0.10
Fiberboard, $\frac{1}{2}$-in. (13 mm)	0.75	0.04

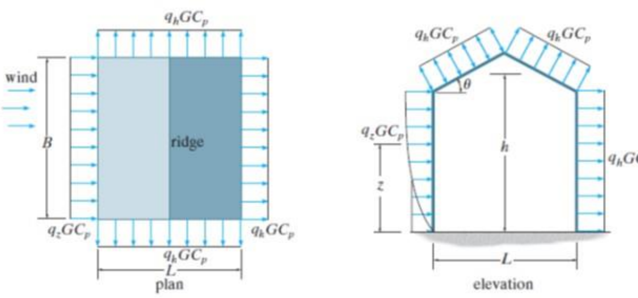
*Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures,* ASCE/SEI 7-10.

TABLE 1–4 Minimum Live Loads*

Occupancy or Use	Live Load psf	Live Load kN/m^2	Occupancy or Use	Live Load psf	Live Load kN/m^2
Assembly areas and theaters			Residential		
Fixed seats	60	2.87	Dwellings (one- and two-family)	40	1.92
Movable seats	100	4.79	Hotels and multifamily houses		
Garages (passenger cars only)	50	2.40	Private rooms and corridors	40	1.92
Office buildings			Public rooms and corridors	100	4.79
Lobbies	100	4.79	Schools		
Offices	50	2.40	Classrooms	40	1.92
Storage warehouse			Corridors above first floor	80	3.83
Light	125	6.00			
Heavy	250	11.97			

*Reproduced with permission from *Minimum Design Loads for Buildings and Other Structures,* ASCE/SEI 7-10.





Simple Example:

EXAMPLE 1.1

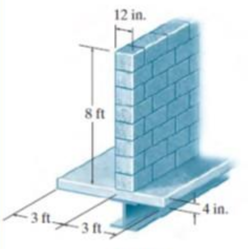


Fig. 1–8

The floor beam in Fig. 1–8 is used to support the 6-ft width of a lightweight plain concrete slab having a thickness of 4 in. The slab serves as a portion of the ceiling for the floor below, and therefore its bottom is coated with plaster. Furthermore, an 8-ft-high, 12-in.-thick lightweight solid concrete block wall is directly over the top flange of the beam. Determine the loading on the beam measured per foot of length of the beam.

SOLUTION

Using the data in Tables 1–2 and 1–3, we have

Concrete slab:	$[8\ \text{lb}/(\text{ft}^2\cdot\text{in.})](4\ \text{in.})(6\ \text{ft}) =$	192 lb/ft
Plaster ceiling:	$(5\ \text{lb}/\text{ft}^2)(6\ \text{ft}) =$	30 lb/ft
Block wall:	$(105\ \text{lb}/\text{ft}^3)(8\ \text{ft})(1\ \text{ft}) =$	840 lb/ft
Total load		1062 lb/ft = 1.06 k/ft

Ans.

Here the unit k stands for "kip," which symbolizes kilopounds. Hence, 1 k = 1000 lb.

1.3 Theory of Structural Analysis Classification

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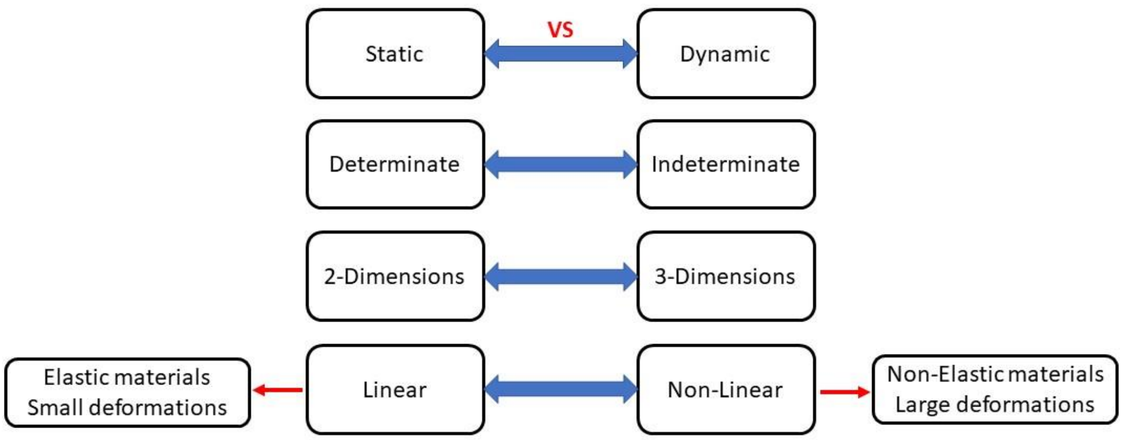
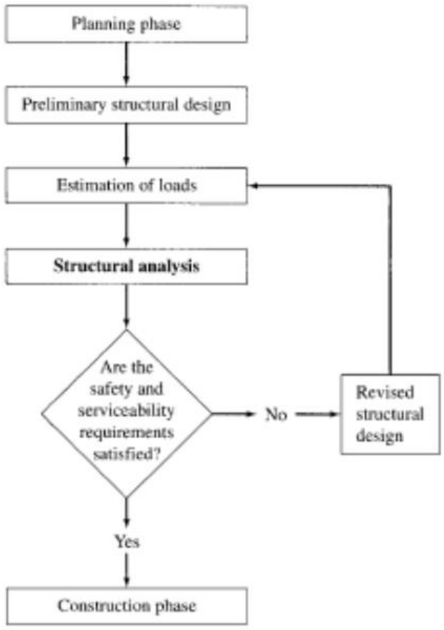
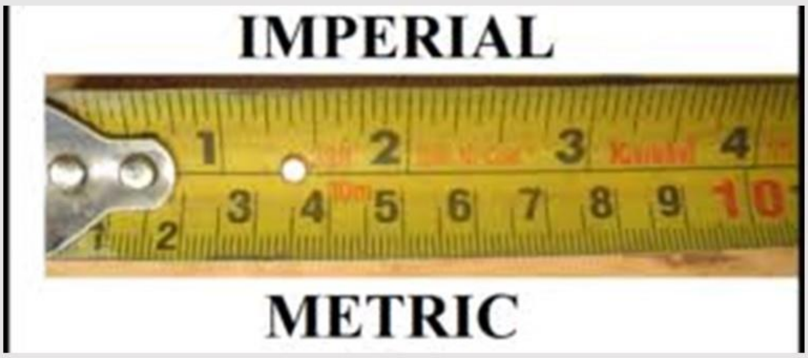


Phases of a typical engineering project



1.4 Units



S.I (System International) Units: m, N, kg, sec

Imperial System Unites: ft, lb, slug, sec

MPa = 10^6 Pa = 10^6 N/mm^2

= 10^6 N/10^6 mm^2 = N/mm^2

Conversion Factors
in = 25.4 mm
m = 3.28 ft
lb = 2.24 N
Kg = 9.81 N

Example:

N/mm^2 → psi (lb/in^2):

$$\frac{N}{mm^2} = \frac{N \times \frac{1}{2.24} \times \frac{lb}{N}}{mm^2 \times \left(\frac{1}{25.4}\right)^2 \times \frac{in^2}{mm^2}} = \frac{(25.4)^2}{2.24}\ \frac{lb}{in^2}$$

$$= 145\frac{lb}{in^2} = 145\ \text{psi}$$

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Example:

Pcf (lb/ft^3)→ kN/m^3:

$$\frac{lb}{ft^3} = \frac{\frac{2.24}{1000}N}{\left(\frac{1}{3.28}\right)^3} = 0.079\frac{kN}{m^3}$$

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1.5 Multiplication Factors

10^{3} = kilo
10^{6} = mega
10^{9} = giga
10^{12} = tetra
10^{-3} = milli
10^{-6} = micro
10^{-9} = nano

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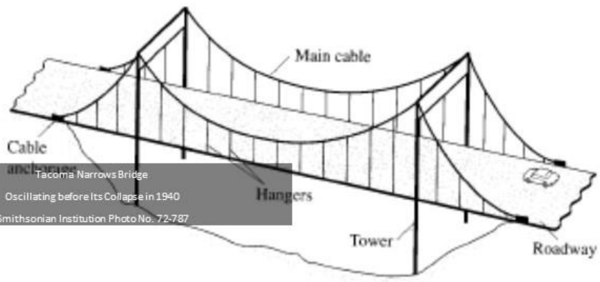
1.6 Idealization of a Structure and Loading



Tacoma Narrows Bridge
Oscillating before Its Collapse in 1940
Smithsonian Institution Photo No. 72-787

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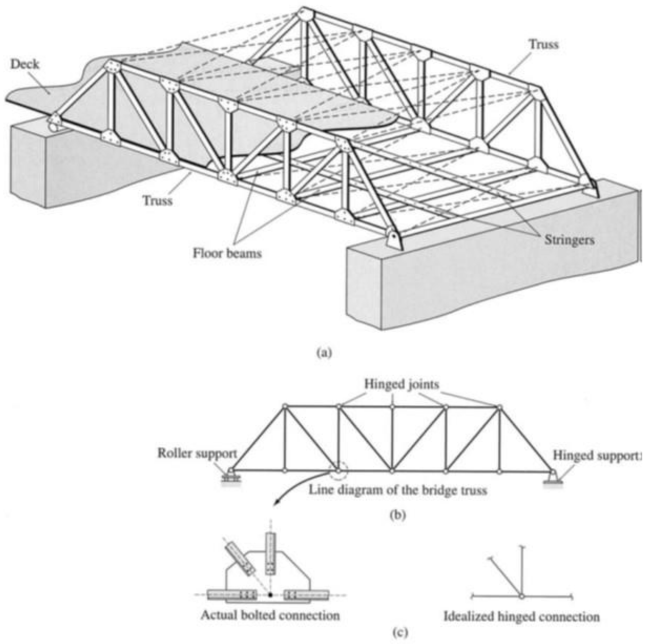

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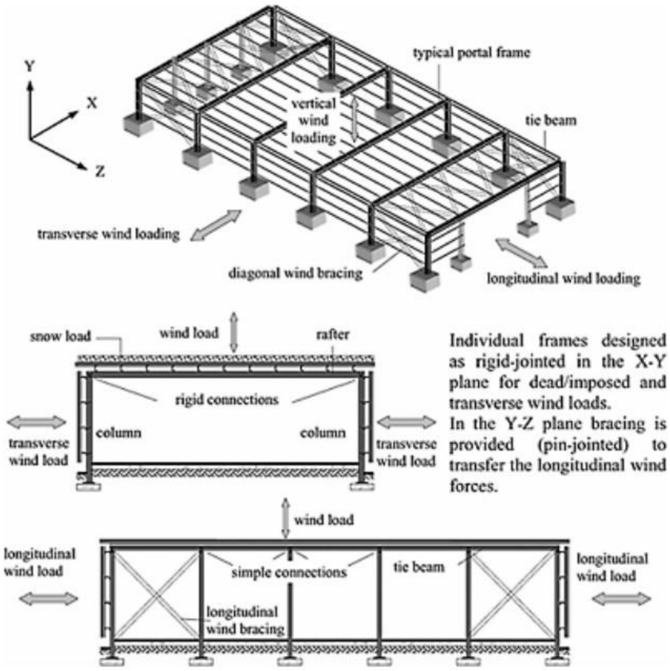

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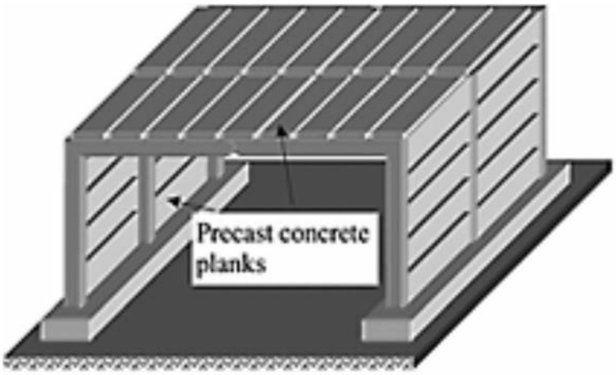

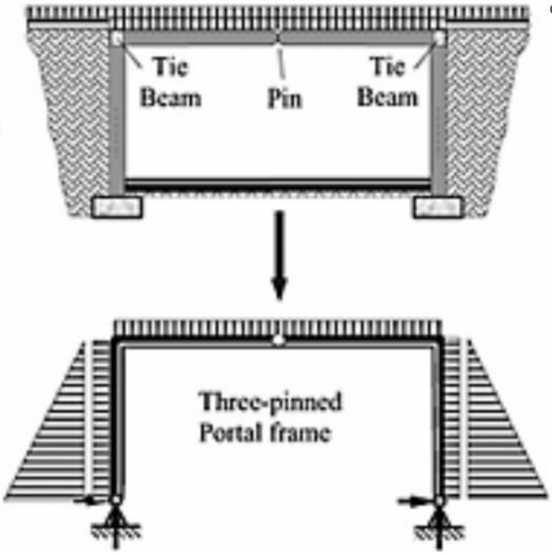

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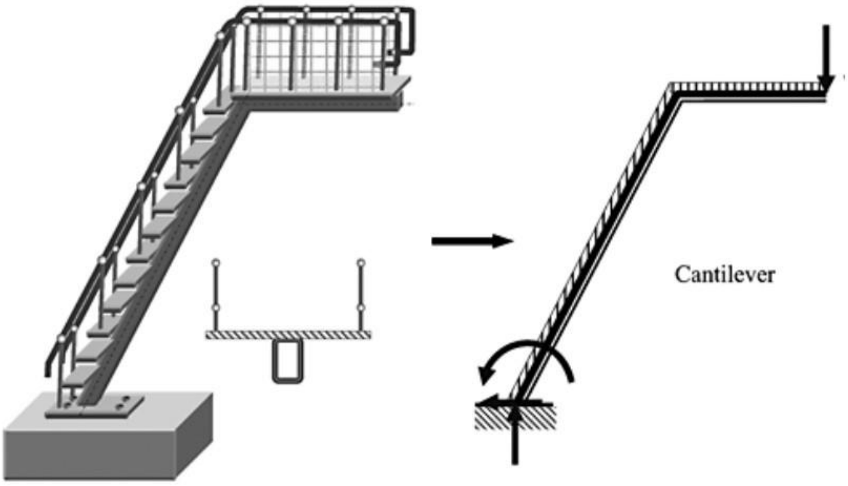

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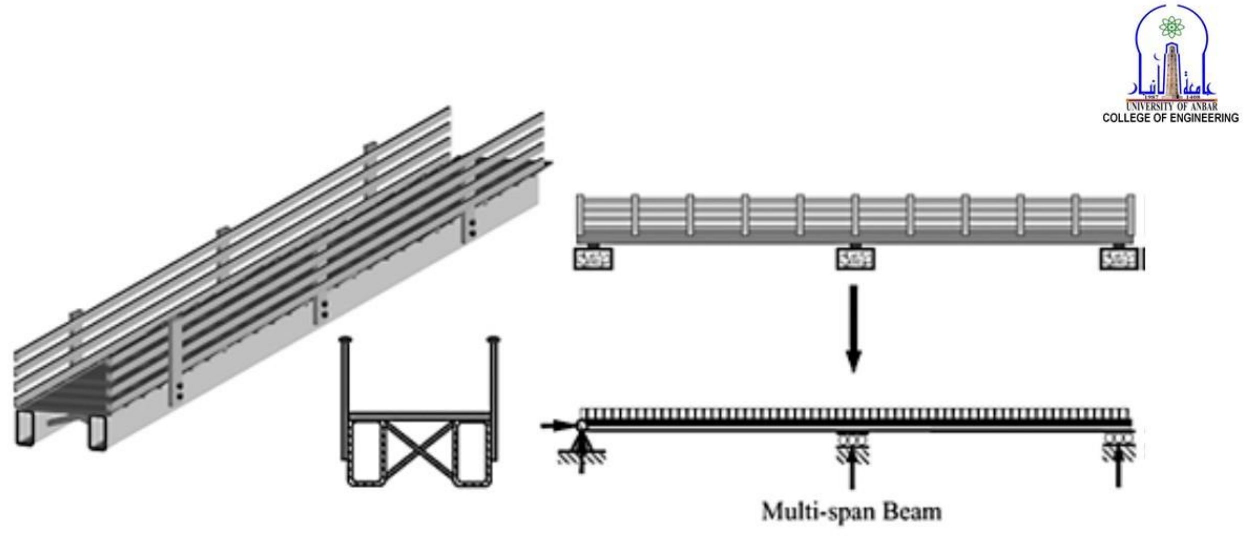
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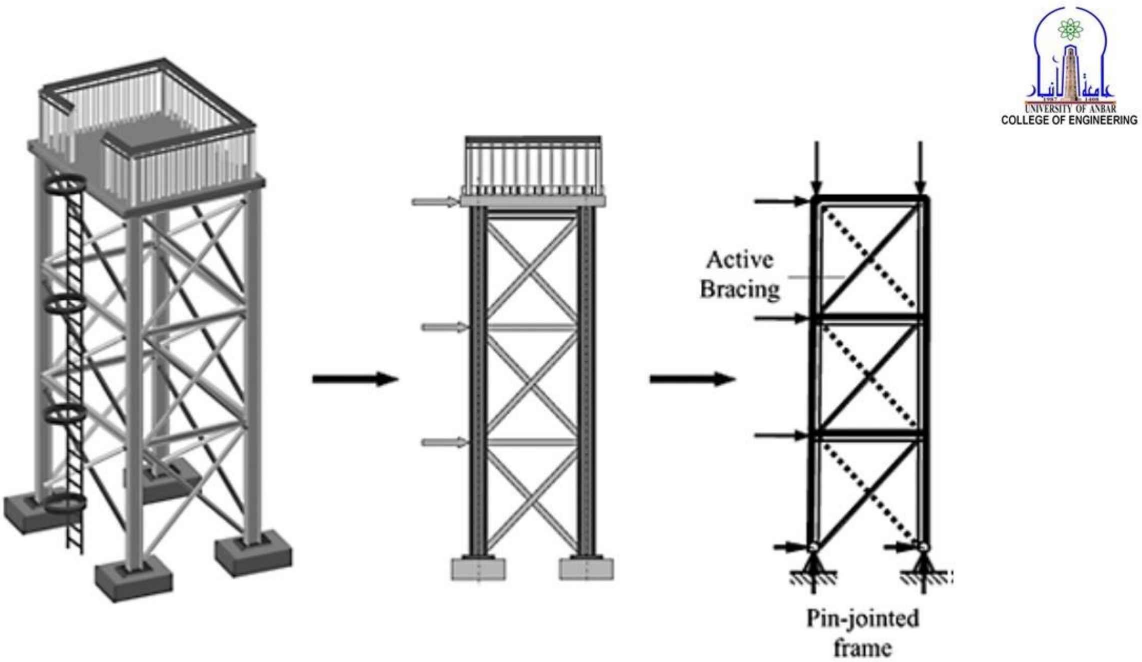
Multi-span Beam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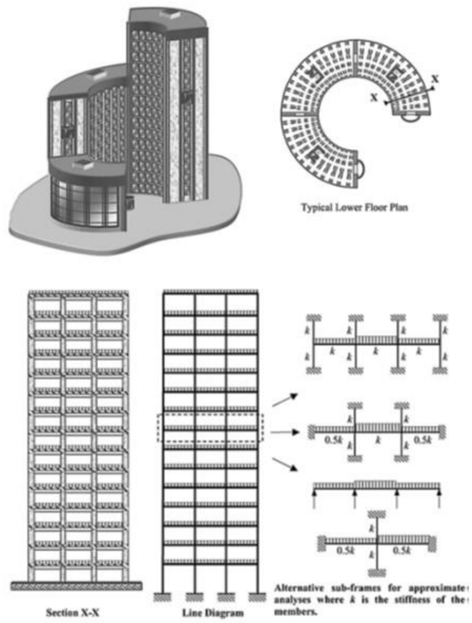
Active
Bracing
Pin-jointed
frame



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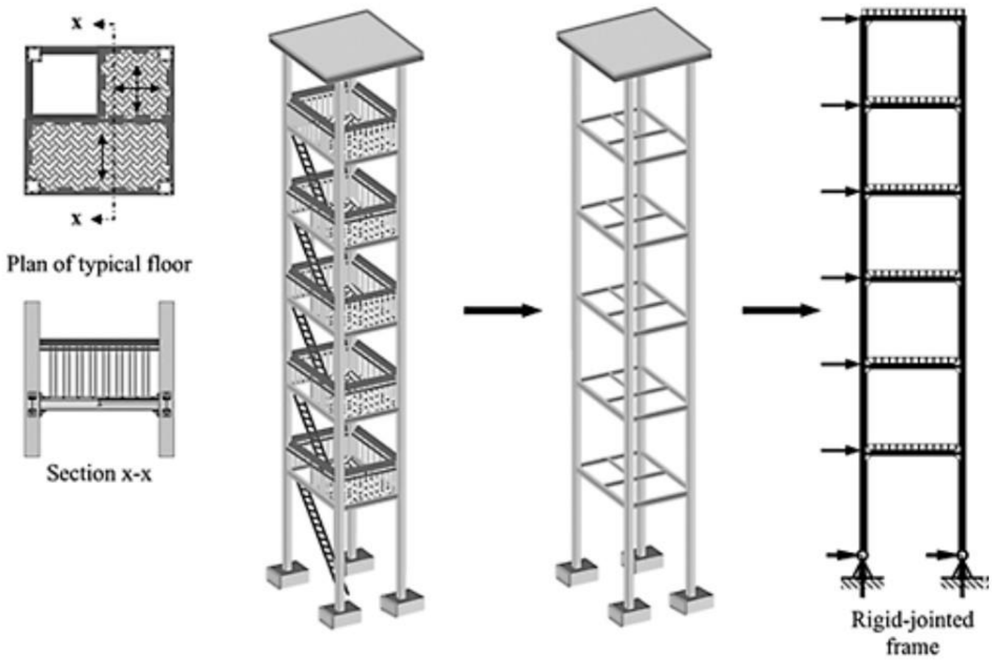

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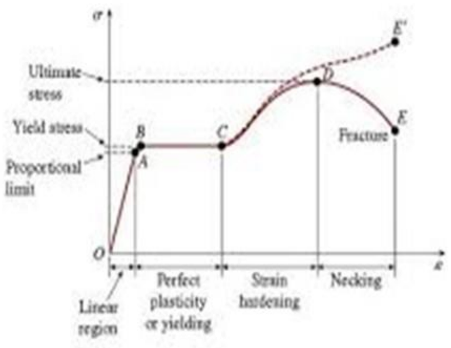
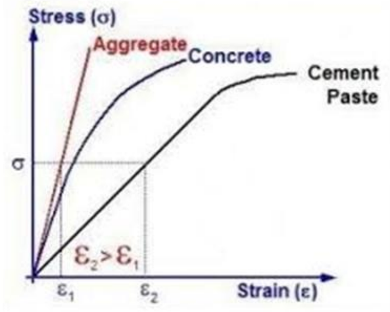
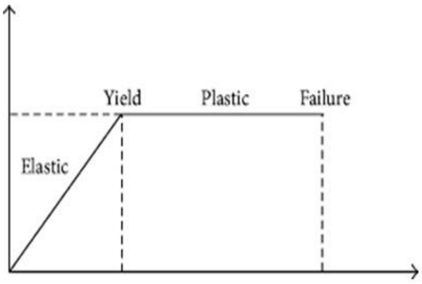
1.7 Principles of Elastic Structural Analysis



Steel



Concrete



Idealized

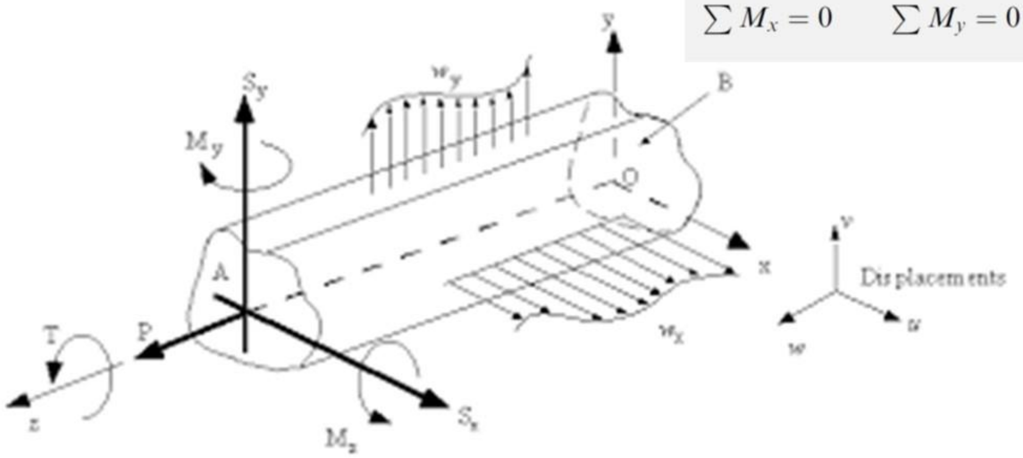
Principles:

1. Linear & Elastic
2. Small displacement principle
3. Superposition
4. Equilibrium

1.8 Equilibrium and Force Systems

A- Three-dimensional equilibrium equations:

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$



1.8 Equilibrium and Force Systems

B- Two-dimensional equilibrium equations:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_z = 0$$

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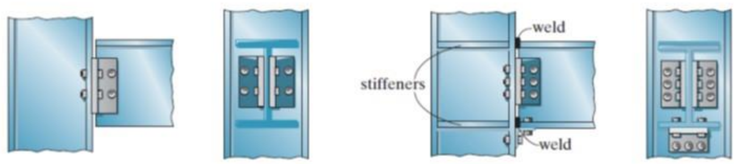
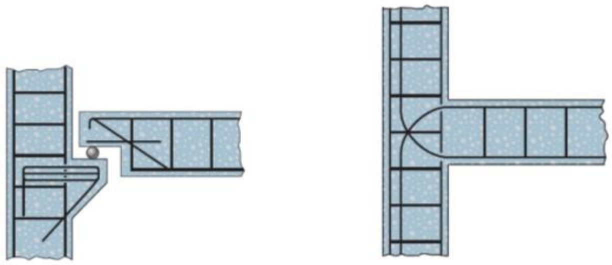
C- Real-Life Supports:



typical "pin-supported" connection (metal)
(a)

typical "fixed-supported" connection (metal)
(b)



typical "roller-supported" connection (concrete)
(a)

typical "fixed-supported" connection (concr
(b)

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D- Idealized Supports:

Category	Type of support	Symbolic representation	Reactions	Number of unknowns
I	Roller	or	R or θ R	1 The reaction force R acts perpendicular to the supporting surface and may be directed either into or away from the structure. The magnitude of R is the unknown.
	Rocker		R	
	Link	θ	R θ	1 The reaction force R acts in the direction of the link and may be directed either into or away from the structure. The magnitude of R is the unknown.
II	Hinge	or	R_x θ R R_y or R_x R_y	2 The reaction force R may act in any direction. It is usually convenient to represent R by its rectangular components, R_x and R_y. The magnitudes of R_x and R_y are the two unknowns.
III	Fixed		R_x M R_y	3 The reactions consist of two force components R_x and R_y and a couple of moment M. The magnitudes of R_x, R_y, and M are the three unknowns.

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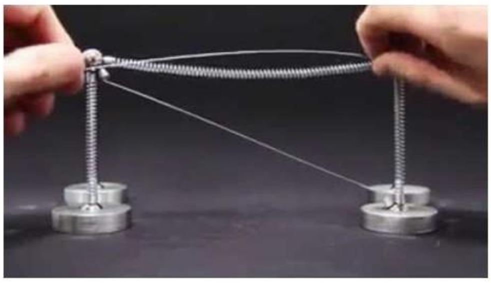

1.9 Stability and Indeterminacy of Structures

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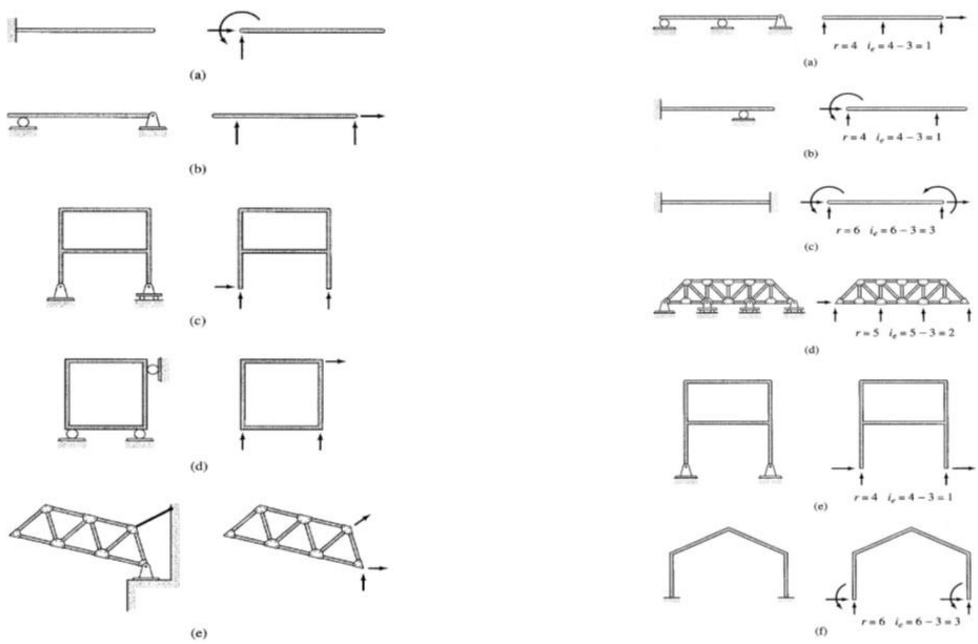
1. Statically determinate structures: Structures that can be analysed using equilibrium equations only.

2. Statically indeterminate structures: Structures can not be analysed using equilibrium equations only.

3. Redundant forces: The extra reactions that exceeds and can not be found by equilibrium equations.



Determinate

Indeterminate

Degree of Indeterminacy:

I.D = No. of Unknowns – No. of Equations

$$I.D = NUK - NEQ$$

(a) Beams:

NUK = Reactions (R)
NEQ = 3+C

$r = 3n$, statically determinate
$r > 3n$, statically indeterminate

C = No. of Conditional Equations

$$I.D = NUK - NEQ = R-(3+C)$$

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Example:

(a)

$r = 3, n = 1, 3 = 3(1)$

Statically determinate

$I.D = R-(3+C) = 3-(3+0) = 0$ → Determinate

(b)

$r = 5, n = 1, 5 > 3(1)$

Statically indeterminate to the second degree

$I.D = R-(3+C) = 5-(3+0) = 2$ → Indeterminate 2nd Degree

(c)

$r = 6, n = 2, 6 = 3(2)$

Statically determinate

$I.D = R-(3+C) = 4-(3+1) = 0$ → Determinate

(d)

$r = 10, n = 3, 10 > 3(3)$

Statically indeterminate to the first degree

$I.D = R-(3+C) = 6-(3+2) = 1$ → Indeterminate 1st Degree

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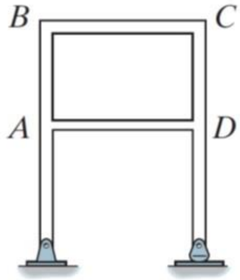
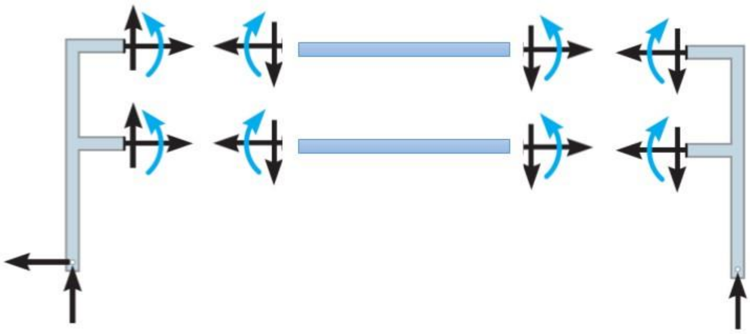
(b) Frames: Method-1 and 2



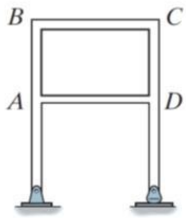
NUK = 6m+R
NEQ = 3m+3j+C

C = No. of Conditional Equations

I.D = NUK – NEQ
= 3m+R-(3j+C)



Example:

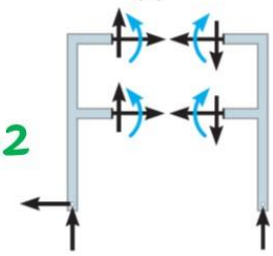


(a)

Method-1

I.D = 3m+R-(3j+C)
I.D = [3(6)+3]–[3(6)+0]
I.D = 21-18 = 3

Method-2



$r = 9$, $n = 2$, $9 > 6$,
Statically indeterminate to the third degree *Ans.*

(a)

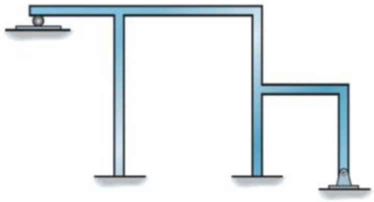
Example:

Method-1

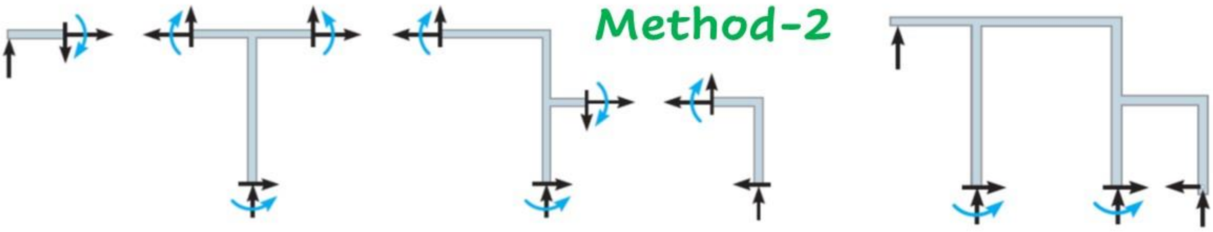
I.D = 3m+R-(3j+C)

I.D = [3(7)+9]–[3(8)+0]

I.D = 30-24 = 6



Method-2



$r = 9$, $n = 1$, $9 > 3$,
Statically indeterminate to the sixth degree *Ans.*

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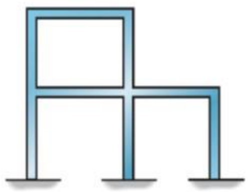
Example:

Method-1

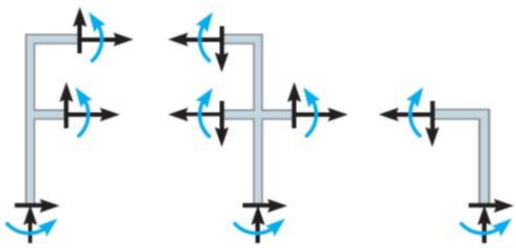
I.D = 3m+R-(3j+C)

I.D = [3(8)+9]–[3(8)+0]

I.D = 33-24 = 9



Method-2



$r = 18$, $n = 3$, $18 > 9$,
Statically indeterminate to the ninth degree *Ans.*

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(b) Trusses:

NUK = m + R
NEQ = 2j

I.D = NUK – NEQ
= m+R-2j

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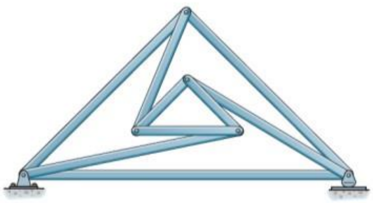
Examples:

I.D = m+R-2j
I.D =19 + 3 - 2(11)
I.D = 22-22 = 0 → **Determinate**

I.D = m+R-2j
I.D =9 + 3 - 2(6)
I.D = 12-12 = 0 → **Determinate**



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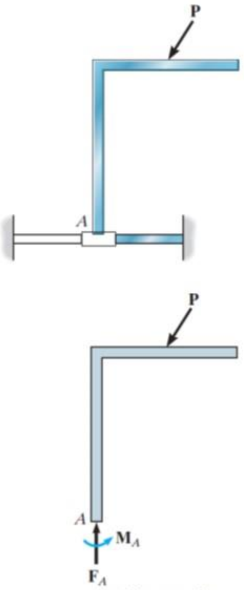
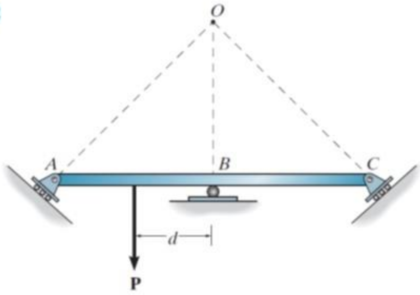
Stability

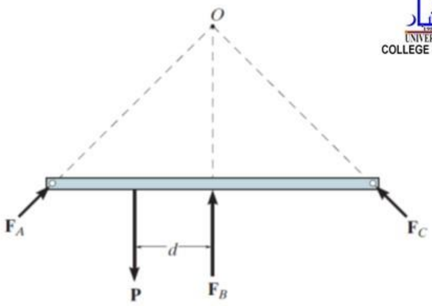
In general, when the equations of static equilibrium are satisfied, the structure is at rest and would say to be a **STABLE** structure. When the structure, or any part of it, cannot satisfy the equilibrium equations, it is said to be **UNSTABLE**!

Examples of Externally Unstable Structures

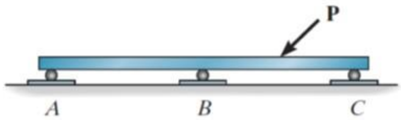


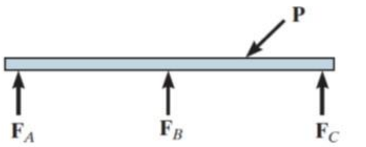
partial constraints





concurrent reactions





parallel reactions

Summary

For Beams:

$R<C+3 \rightarrow$ Unstable

$R>C+3 \rightarrow$ Stable Indeterminate

$R=C+3 \rightarrow$ Stable determinate

For Frames:

$3M+R<3j+C \rightarrow$ Unstable

$3M+R>3j+C \rightarrow$ Stable Indeterminate

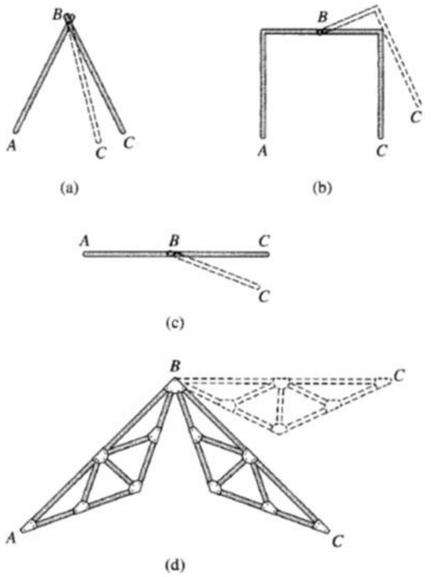
$3M+R=3j+C \rightarrow$ Stable determinate

For Trusses:

$M+R<2j \rightarrow$ Unstable

$M+R>2j \rightarrow$ Stable Indeterminate

$M+R=2j \rightarrow$ Stable determinate



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Unit-2

Statically Determinate Beams and Frames

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2.1 Beams

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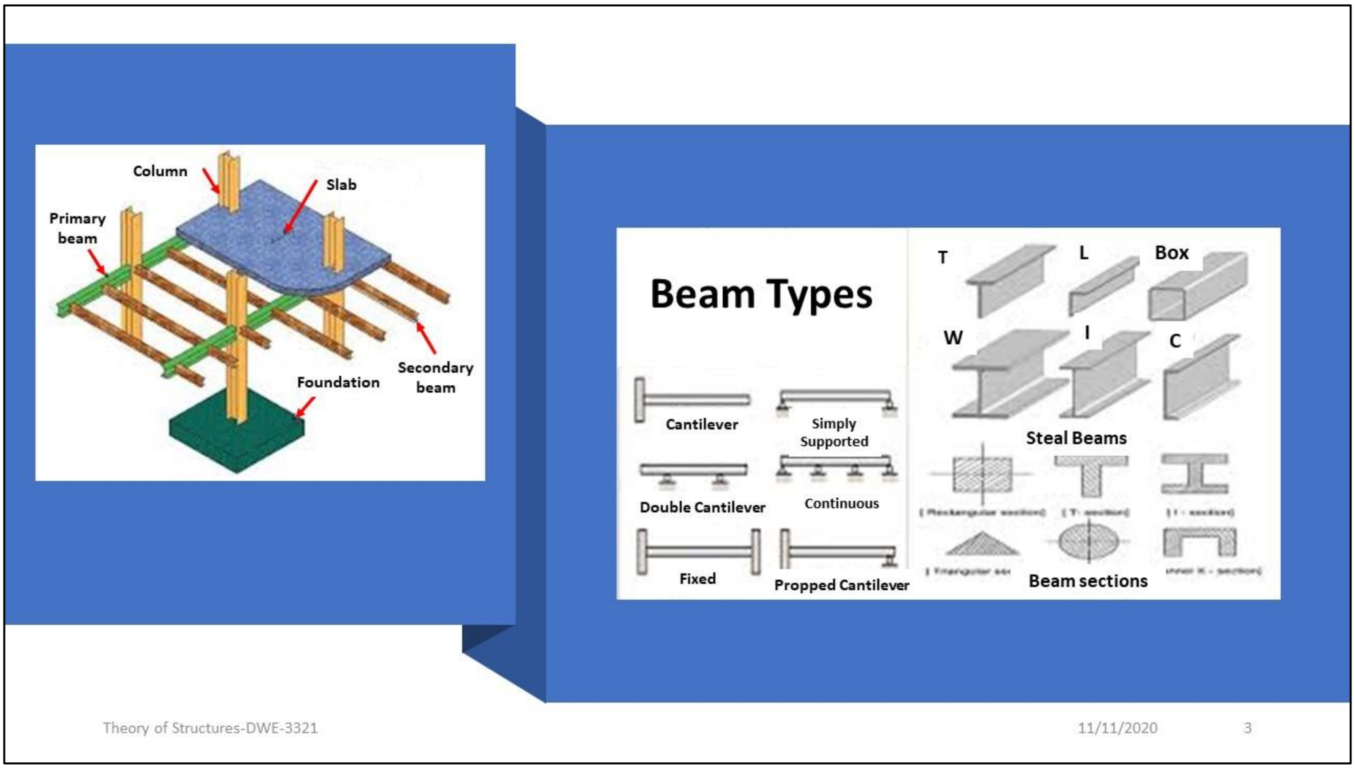
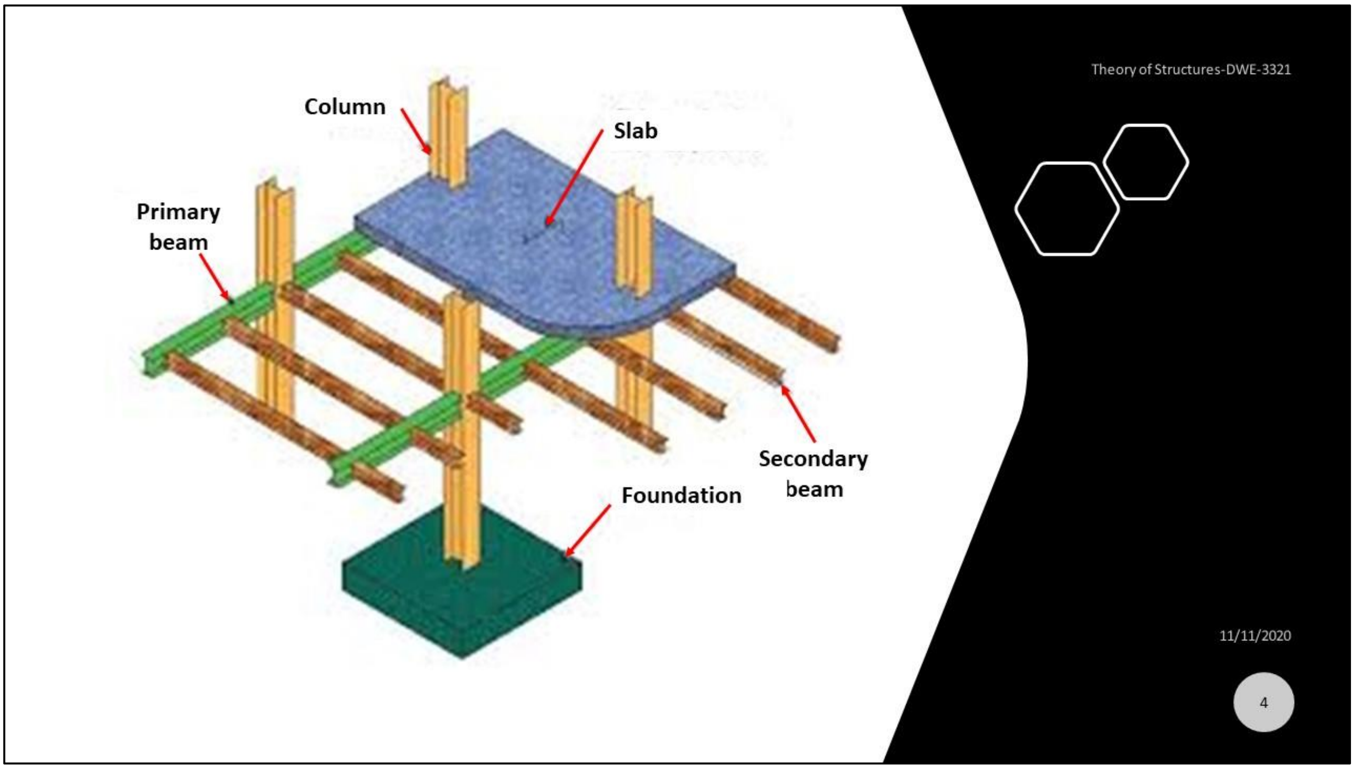
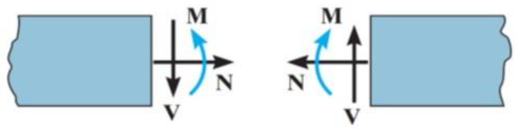

Column
Slab
Primary beam
Secondary beam
Foundation
Beam Types
Cantilever
Simply Supported
Double Cantilever
Continuous
Fixed
Propped Cantilever
T
L
Box
W
I
C
Steal Beams
Beam sections
Theory of Structures-DWE-3321
11/11/2020
3


Theory of Structures-DWE-3321
Column
Slab
Primary beam
Secondary beam
Foundation
11/11/2020
4

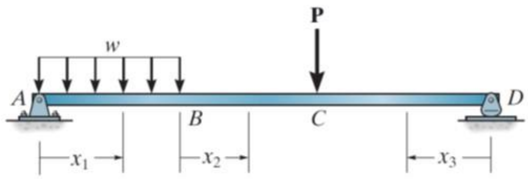
Internal Loadings Developed in Structural Members

Structural members subjected to planar loads support an internal normal force **N**, shear force **V**, and bending moment **M**. To find these values at a specific point in a member, the method of sections must be used. This requires drawing a free-body diagram of a segments of the member, and then applying the three equations of equilibrium.

Always show the three internal loadings on the section in their positive directions.



The internal shear and moment can be expressed as a function of x along the member by establishing the origin at a fixed point (normally at the left end of the member, and then using the method of sections, where the section is made a distance x from the origin). For members subjected to several loads, different x coordinates must extend between the loads.



Shear and moment diagrams for structural members can be drawn by plotting the shear and moment functions. They also can be plotted using the two graphical relationships.

$$\frac{dV}{dx} = w(x)$$

$$\left.\begin{array}{r}\text{Slope of}\\ \text{Shear Diagram}\end{array}\right\} = \left\{\begin{array}{l}\text{Intensity of}\\ \text{Distributed Load}\end{array}\right.$$

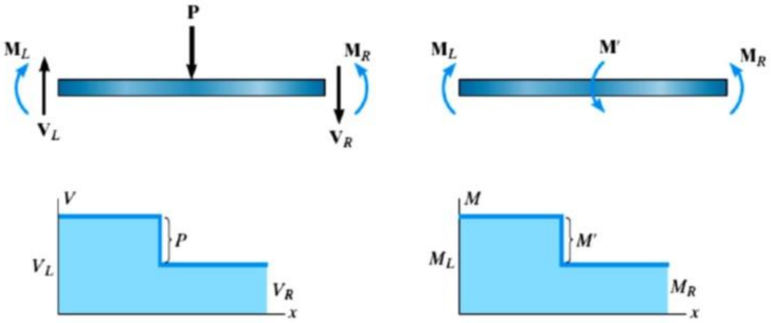
$$\frac{dM}{dx} = V$$

$$\left.\begin{array}{r}\text{Slope of}\\ \text{Moment Diagram}\end{array}\right\} = \{\text{Shear}$$

Note that a point of zero shear locates the point of maximum moment since:

$$V = dM/dx = 0$$

A force acting downward on the beam will cause the shear diagram to jump downwards, and a counterclockwise couple moment will cause the moment diagram to jump downwards.



Using the method of superposition, the moment diagrams for a member can be represented by a series of simpler shapes. The shapes represent the moment diagram for each of the separate loadings. The resultant moment diagram is then the algebraic addition of the separate diagrams.

EXAMPLE 1

Draw the shear and moment diagrams for the beam in Fig. 4–12*a*.

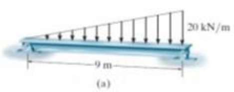
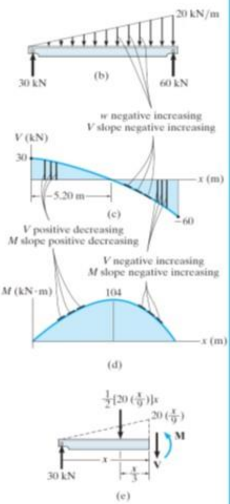


Fig. 4-12



SOLUTION

Support Reactions. The reactions have been calculated and are shown on the free-body diagram of the beam, Fig. 4–12*b*.

Shear Diagram. The end points $x = 0$, $V = +30$ kN and $x = 9$ m, $V = -60$ kN are first plotted. Note that the shear diagram *starts* with zero slope since $w = 0$ at $x = 0$, and ends with a slope of $w = -20$ kN/m.

The point of zero shear can be found by using the method of sections from a beam segment of length x, Fig. 4–12*e*. We require $V = 0$, so that

$$+\uparrow \Sigma F_y = 0; \quad 30 - \frac{1}{2}\left[20\left(\frac{x}{9}\right)\right]x = 0 \quad x = 5.20 \text{ m}$$

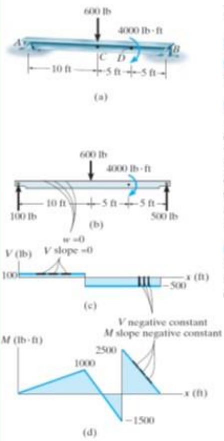
Moment Diagram. For $0 < x < 5.20$ m the value of shear is positive but decreasing and so the slope of the moment diagram is also positive and decreasing ($dM/dx = V$). At $x = 5.20$ m, $dM/dx = 0$. Likewise for $5.20 \text{ m} < x < 9$ m, the shear and so the slope of the moment diagram are negative increasing as indicated.

The maximum value of moment is at $x = 5.20$ m since $dM/dx = V = 0$ at this point, Fig. 4–12*d*. From the free-body diagram in Fig. 4–12*e* we have

$$\circlearrowleft + \Sigma M_S = 0; \quad -30(5.20) + \frac{1}{2}\left[20\left(\frac{5.20}{9}\right)\right](5.20)\left(\frac{5.20}{3}\right) + M = 0$$

$$M = 104 \text{ kN}\cdot\text{m}$$

EXAMPLE 2



Draw the shear and moment diagrams for the beam shown in Fig. 4–13*a*.

SOLUTION

Support Reactions. The reactions are calculated and indicated on the free-body diagram.

Shear Diagram. The values of the shear at the end points A ($V_A = +100$ lb) and B ($V_B = -500$ lb) are plotted. At C the shear is *discontinuous* since there is a *concentrated force* of 600 lb there. The value of the shear just to the right of C can be found by sectioning the beam at this point. This yields the free-body diagram shown in equilibrium in Fig. 4–13*e*. This point ($V = -500$ lb) is plotted on the shear diagram. Notice that no jump or discontinuity in shear occurs at D, the point where the 4000-lb·ft couple moment is applied, Fig. 4–13*b*.

Moment Diagram. The moment at each end of the beam is zero, Fig. 4–13*d*. The value of the moment at C can be determined by the method of sections, Fig. 4–13*e*, or by finding the area under the shear diagram between A and C. Since $M_A = 0$,

$$M_C = M_A + \Delta M_{AC} = 0 + (100 \text{ lb})(10 \text{ ft})$$
$$M_C = 1000 \text{ lb}\cdot\text{ft}$$

Also, since $M_C = 1000$ lb·ft, the moment at D is

$$M_D = M_C + \Delta M_{CD} = 1000 \text{ lb}\cdot\text{ft} + (-500 \text{ lb})(5 \text{ ft})$$
$$M_D = -1500 \text{ lb}\cdot\text{ft}$$

A jump occurs at point D due to the couple moment of 4000 lb·ft. The method of sections, Fig. 4–13*f*, gives a value of +2500 lb·ft just to the right of D.

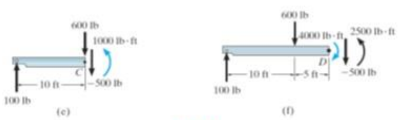
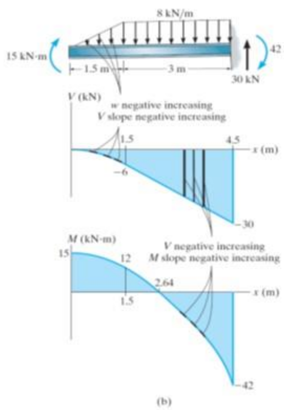


Fig. 4-13

EXAMPLE 3

Draw the shear and moment diagrams for each of the beams shown in Fig. 4–14.



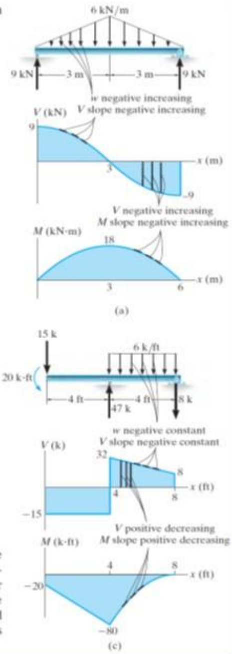


Fig. 4–14

SOLUTION

In each case the support reactions have been calculated and are shown in the top figures. Following the techniques outlined in the previous examples, the shear and moment diagrams are shown under each beam. Carefully notice how they were established, based on the slope and moment, where $dV/dx = w$ and $dM/dx = V$. Calculated values are found using the method of sections or finding the areas under the load or shear diagrams.

EXAMPLE 4

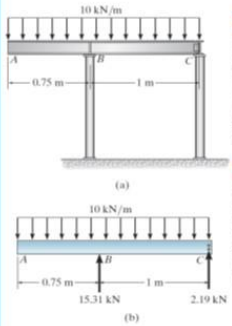
The beam shown in the photo is used to support a portion of the overhang for the entranceway of the building. The idealized model for the beam with the load acting on it is shown in Fig. 4–15*a*. Assume *B* is a roller and *C* is pinned. Draw the shear and moment diagrams for the beam.

SOLUTION

Support Reactions. The reactions are calculated in the usual manner. The results are shown in Fig. 4–15*b*.

Shear Diagram. The shear at the ends of the beam is plotted first, i.e., $V_A = 0$ and $V_C = -2.19$ kN, Fig. 4–15*c*. To find the shear to the left of *B* use the method of sections for segment *AB*, or calculate the area under the distributed loading diagram, i.e., $\Delta V = V_B - 0 = -10(0.75)$, $V_{B^-} = -7.50$ kN. The support reaction causes the shear to jump up $-7.50 + 15.31 = 7.81$ kN. The point of zero shear can be determined from the slope -10 kN/m, or by proportional triangles, $7.81/x = 2.19/(1 - x)$, $x = 0.781$ m. Notice how the *V* diagram follows the negative slope, defined by the constant negative distributed loading.

Moment Diagram. The moment at the end points is plotted first, $M_A = M_C = 0$, Fig. 4–15*d*. The values of -2.81 and 0.239 on the moment diagram can be calculated by the method of sections, or by finding the areas under the shear diagram. For example, $\Delta M = M_B - 0 = \frac{1}{2}(-7.50)(0.75) = -2.81$, $M_B = -2.81$ kN·m. Likewise, show that the maximum positive moment is 0.239 kN·m. Notice how the *M* diagram is formed, by following the slope, defined by the *V* diagram.



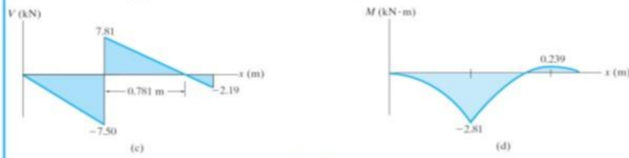
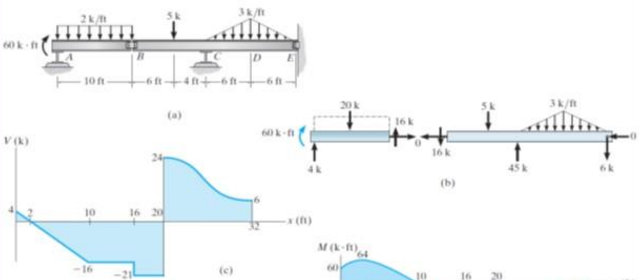


Fig. 4–15

EXAMPLE 5

Draw the shear and moment diagrams for the compound beam shown in Fig. 4–16*a*. Assume the supports at *A* and *C* are rollers and *B* and *E* are pin connections.



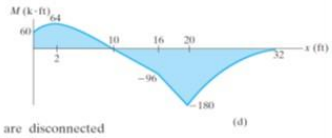


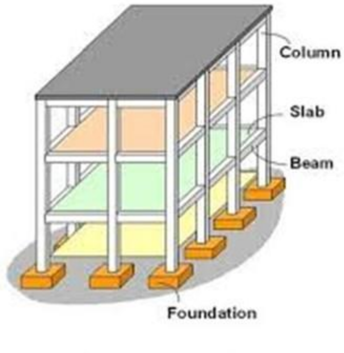
Fig. 4–16

SOLUTION

Support Reactions. Once the beam segments are disconnected from the pin at *B*, the support reactions can be calculated as shown in Fig. 4–16*b*.

Shear Diagram. As usual, we start by plotting the end shear at *A* and *E*, Fig. 4–16*c*. The shape of the *V* diagram is formed by following its slope, defined by the loading. Try to establish the values of shear using the appropriate areas under the load diagram (*w* curve) to find the change in shear. The zero value for shear at $x = 2$ ft can either be found by proportional triangles, or by using statics, as was done in Fig. 4–12*e* of Example 4–8.

Moment Diagram. The end moments $M_A = 60$ k·ft and $M_E = 0$ are plotted first, Fig. 4–16*d*. Study the diagram and note how the various curves are established using $dM/dx = V$. Verify the numerical values for the peaks using statics or by calculating the appropriate areas under the shear diagram to find the change in moment.



Typical RC Frame Building

2.2 Frames

EXAMPl 6

Draw the moment diagram for the tapered frame shown in Fig. 4–17*a*. Assume the support at *A* is a roller and *B* is a pin.

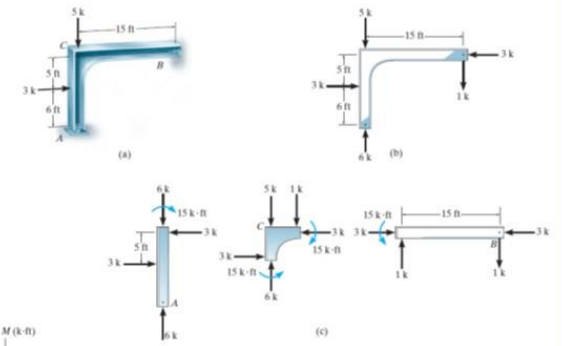
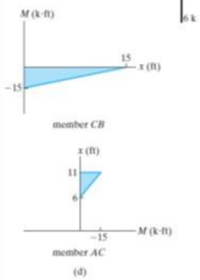


Fig. 4–17



SOLUTION

Support Reactions. The support reactions are shown on the free-body diagram of the entire frame, Fig. 4–17*b*. Using these results, the frame is then sectioned into two members, and the internal reactions at the joint ends of the members are determined, Fig. 4–17*c*. Note that the external 5-k load is shown only on the free-body diagram of the joint at *C*.

Moment Diagram. In accordance with our positive sign convention, and using the techniques discussed in Sec. 4–3, the moment diagrams for the frame members are shown in Fig. 4–17*d*.

EXAMPLE 7

Draw the shear and moment diagrams for the frame shown in Fig. 4–18*a*. Assume *A* is a pin, *C* is a roller, and *B* is a fixed joint. Neglect the thickness of the members.

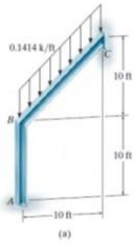
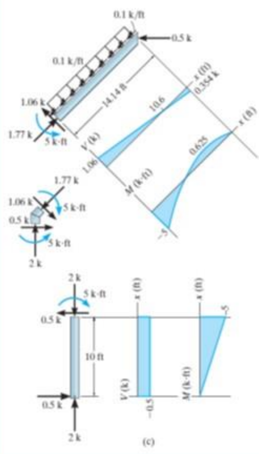
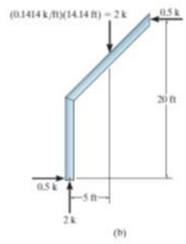


Fig. 4–18

SOLUTION

Notice that the distributed load acts over a length of $10\text{ ft}\sqrt{2} = 14.14\text{ ft}$. The reactions on the entire frame are calculated and shown on its free-body diagram, Fig. 4–18*b*. From this diagram the free-body diagrams of each member are drawn, Fig. 4–18*c*. The distributed loading on *BC* has components along *BC* and perpendicular to its axis of $(0.1414\text{ k/ft})\cos 45^\circ = (0.1414\text{ k/ft})\sin 45^\circ = 0.1\text{ k/ft}$ as shown. Using these results, the shear and moment diagrams are also shown in Fig. 4–18*c*.





EXAMPLE 8

Draw the shear and moment diagrams for the frame shown in Fig. 4–19a. Assume A is a pin, C is a roller, and B is a fixed joint.

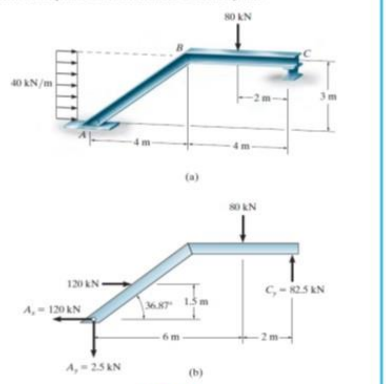
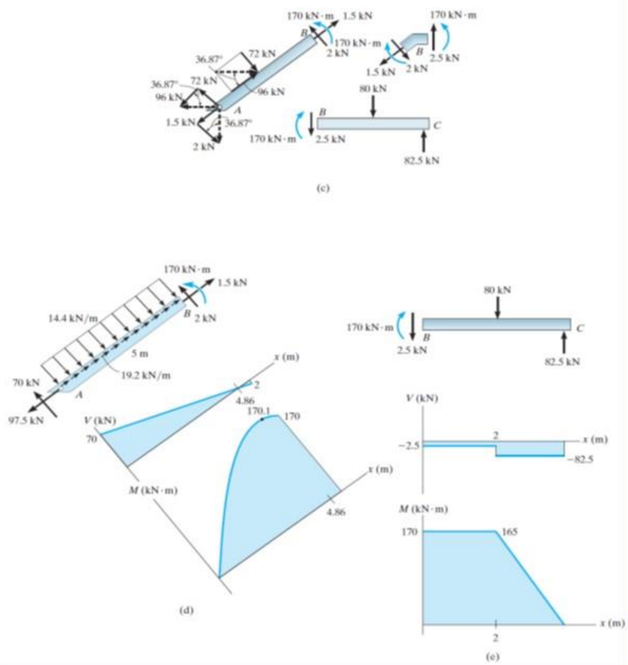


Fig. 4–19

SOLUTION

Support Reactions. The free-body diagram of the entire frame is shown in Fig. 4–19b. Here the distributed load, which represents wind loading, has been replaced by its resultant, and the reactions have been computed. The frame is then sectioned at joint B and the internal loadings at B are determined, Fig. 4–19c. As a check, equilibrium is satisfied at joint B, which is also shown in the figure.

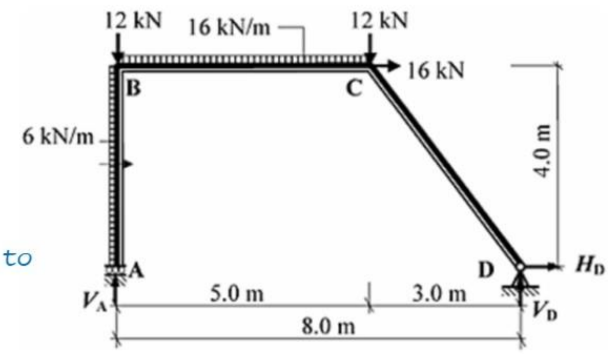
Shear and Moment Diagrams. The components of the distributed load, (72 kN)/(5 m) = 14.4 kN/m and (96 kN)/(5 m) = 19.2 kN/m, are shown on member AB, Fig. 4–19d. The associated shear and moment diagrams are drawn for each member as shown in Figs. 4–19d and 4–19e.



Example -9

An asymmetric portal frame is supported on a roller at A and pinned at support D as shown in Figure below. For the loading indicated:

i) determine the support reactions and,
ii) sketch the axial load, shear force and bending moment diagrams.



Solution:

Apply the three equations of static equilibrium to the force system

$$+\text{ve} \uparrow \Sigma F_y = 0 \qquad V_A - 12.0 - (16.0 \times 5.0) - 12.0 + V_D = 0$$

$$+\text{ve} \rightarrow \Sigma F_x = 0 \qquad (6.0 \times 4.0) + 16.0 + H_D = 0$$

$$+\text{ve} \circlearrowright \Sigma M_A = 0 \qquad (6.0 \times 4.0)(2.0) + (16.0 \times 5.0)(2.5) + (12.0 \times 5.0) + (16.0 \times 4.0) - (V_D \times 8.0) = 0$$

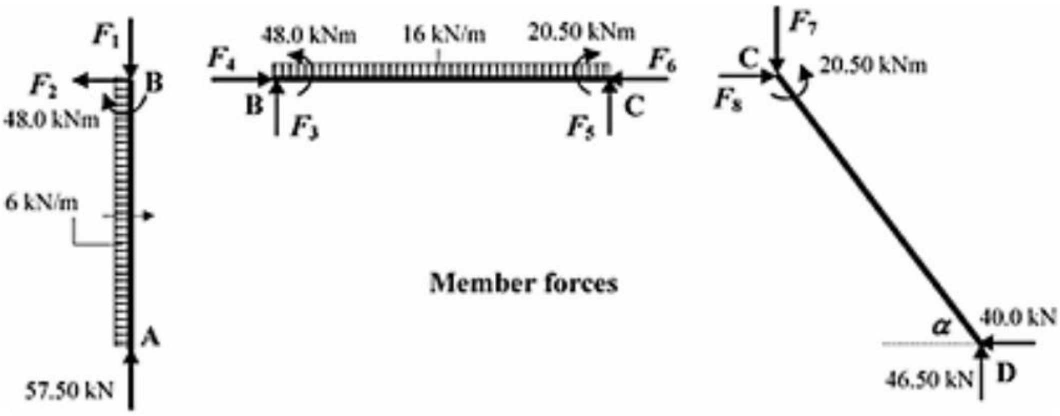
From equation (2): $40.0 + H_D = 0$ $\quad \therefore H_D = -40.0$ kN $\leftarrow$

From equation (3): $372.0 - 8.0V_D = 0$ $\quad \therefore V_D = +46.5$ kN $\uparrow$

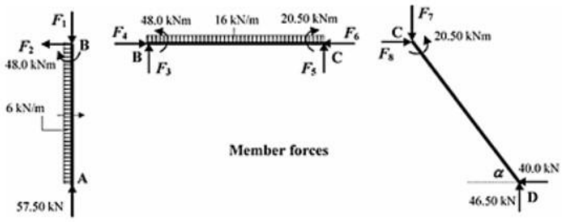
From equation (1): $V_A - 104.0 + 46.5 = 0$ $\quad \therefore V_A = +57.5$ kN $\uparrow$

Assuming positive bending moments induce tension inside the frame:

MB= −(6.0×4.0)(2.0)= −48.0 kN.m
MC= +(46.5×3.0)−(40.0×4.0)= −20.50 kN.m



The values of the end-forces F1 to F8 can be determined by considering the equilibrium of each member and joint in turn.



Consider member AB:

$+ve \uparrow \Sigma F_y = 0 \quad + 57.50 - F_1 = 0 \qquad \therefore F_1 = 57.50 \text{ kN} \downarrow$

$+ve \rightarrow \Sigma F_x = 0 \quad + (6.0 \times 4.0) - F_2 = 0 \qquad \therefore F_2 = 24.0 \text{ kN} \leftarrow$

Consider joint B:

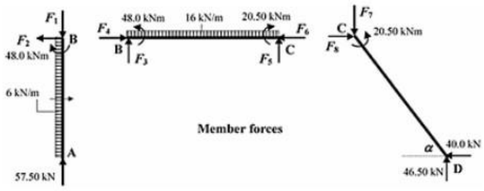
$+ve \uparrow \Sigma F_y = 0$ There is an applied vertical load at joint B = 12 kN $\downarrow$

$- F_1 + F_3 = - 12.0 \qquad \therefore F_3 = 45.50 \text{ kN} \uparrow$

$+ve \rightarrow \Sigma F_x = 0$

$- F_2 + F_4 = 0 \qquad \therefore F_4 = 24.0 \text{ kN} \rightarrow$

Consider member BC:



+ve $\uparrow \Sigma F_y = 0 \quad + 45.5 - (16.0 \times 5.0) + F_5 = 0 \qquad \therefore F_5 = 34.5 \text{ kN} \uparrow$

+ve $\rightarrow \Sigma F_x = 0 \quad + 24.0 - F_6 = 0 \qquad \therefore F_6 = 24.0 \text{ kN} \leftarrow$

Consider member CD:

+ve $\uparrow \Sigma F_y = 0 \quad + 46.5 - F_7 = 0 \qquad \therefore F_7 = 46.5 \text{ kN} \downarrow$

+ve $\rightarrow \Sigma F_x = 0 \quad - 40.0 + F_8 = 0 \qquad \therefore F_8 = 40.0 \text{ kN} \rightarrow$

Check joint C:

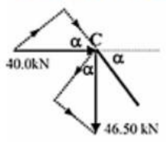
+ve $\uparrow \Sigma F_y$ There is an applied vertical load at joint C = 12 kN $\downarrow$

$+ F_5 - F_7 = + 34.5 - 46.5 = - 12.0$

+ve $\rightarrow \Sigma F_x$ There is an applied horizontal at joint C = 16 kN $\rightarrow$

$- F_6 + F_8 = - 24.0 + 40.0 = + 16.0$

Member CD:



$\alpha = \tan^{-1}(4.0/3.0) = 53.13°$

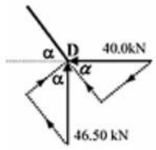
$\text{Cos}\ \alpha = 0.60; \quad \text{Sin}\ \alpha = 0.80$

Assume axial compression to be positive.

At joint C

Axial force = $+ (40.0 \times \text{Cos}\alpha) + (46.50 \times \text{Sin}\alpha) = + 61.2$ kN

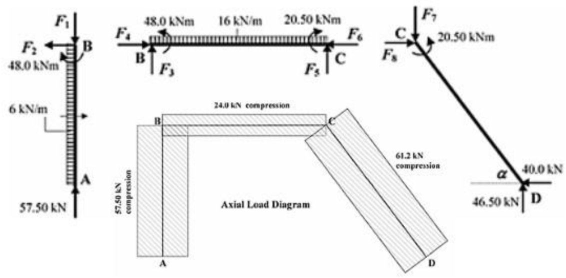
Shear force = $+ (40.0 \times \text{Sin}\alpha) - (46.50 \times \text{Cos}\alpha) = + 4.10$ kN

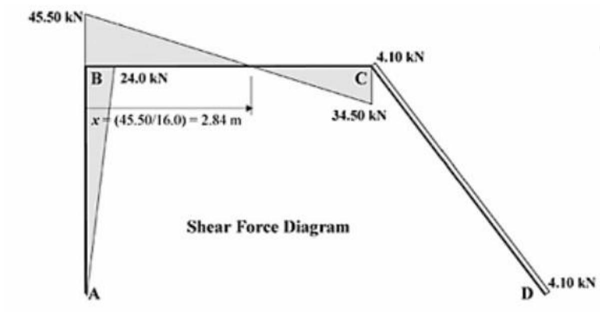


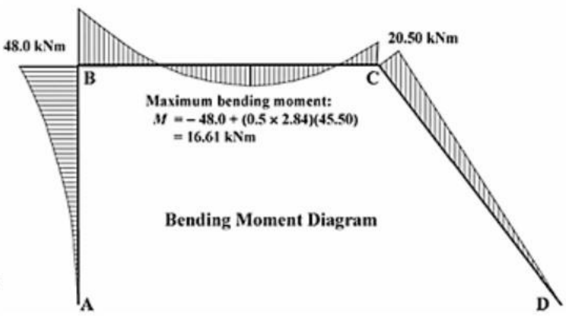
Similarly at joint D

Axial force = + 61.2 kN

Shear force = + 4.10 kN



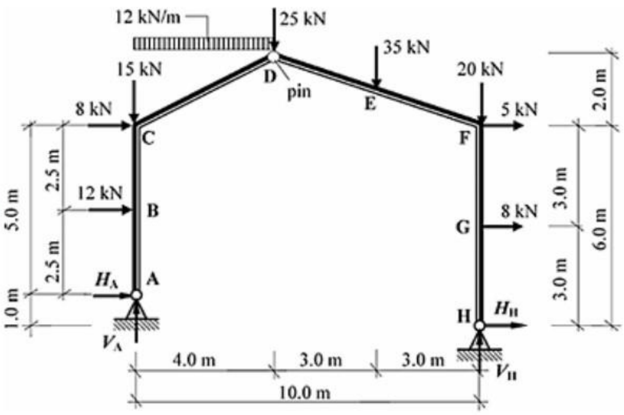




Example -10

A pitched-roof portal frame is pinned at supports A and H and members CD and DEF are pinned at the ridge as shown in Figure 5.6. For the loading indicated:

i) determine the support reactions and
ii) sketch the axial load, shear force and bending moment diagrams.

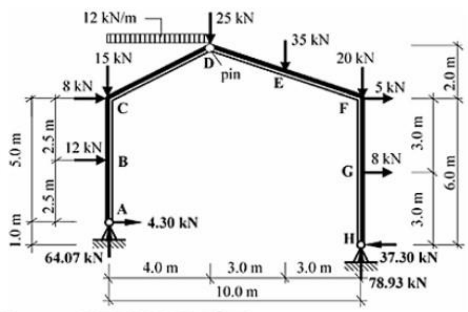


Solution:

Apply the three equations of static equilibrium to the force system in addition to the Σ moments at the pin = 0:

$$+\text{ve} \uparrow \Sigma F_y = 0$$
$$V_A - 15.0 - (12.0 \times 4.0) - 25.0 - 35.0 - 20.0 + V_H = 0$$
$$+\text{ve} \rightarrow \Sigma F_x = 0$$
$$H_A + 12.0 + 8.0 + 5.0 + 8.0 + H_H = 0$$
$$+\text{ve} \circlearrowright \Sigma M_A = 0$$

$$(12.0 \times 2.5) + (8.0 \times 5.0) + (12.0 \times 4.0)(2.0) + (25.0 \times 4.0) + (35.0 \times 7.0)$$
$$+ (20.0 \times 10.0) + (5.0 \times 5.0) + (8.0 \times 2.0) - (H_H \times 1.0) - (V_H \times 10.0) = 0$$
$$+\text{ve} \circlearrowright \Sigma M_{pin} = 0 \quad \text{(right-hand side)}$$
$$+ (35.0 \times 3.0) + (20.0 \times 6.0) - (5.0 \times 2.0) - (8.0 \times 5.0) - (H_H \times 8.0) - (V_H \times 6.0) = 0$$



From Equation (3): $\quad + 752.0 - H_H - 10.0V_H = 0$

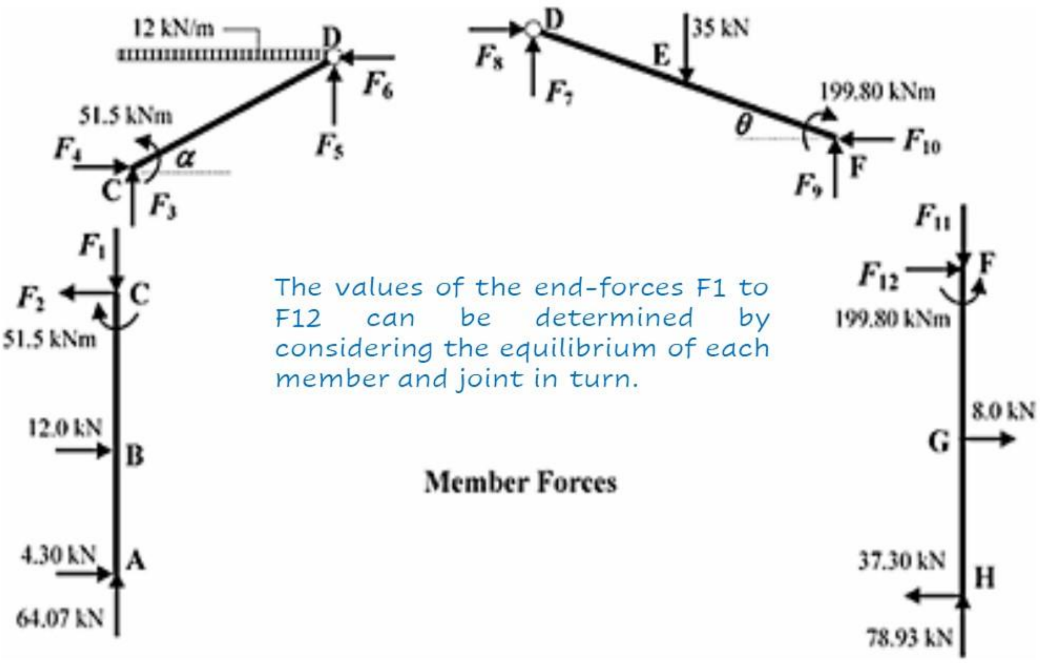
From Equation (4): $\quad + 175.0 - 8.0H_H - 6.0V_H = 0$

Solve equations 3(a) and 3(b) simultaneously: $V_H = +78.93$ kN ↑ $\quad H_H = -37.30$ kN ←

From Equation (2): $\quad H_A + 33.0 + H_H = 0 \qquad H_A = +4.30$ kN →

From Equation (1): $\quad V_A - 143.0 + V_H = 0 \qquad V_A = +64.07$ kN ↑

$M_B = -(4.30 \times 2.5) = -10.75$ kNm
$M_C = -(4.30 \times 5.0) - (12.0 \times 2.5) = -51.50$ kNm
$M_D =$ zero (pin)
$M_E = -(20.0 \times 3.0) + (5.0 \times 1.0) + (8.0 \times 4.0) - (37.3 \times 7.0) + (78.93 \times 3.0)$
$\quad = -47.31$ kNm
$M_F = +(8.0 \times 3.0) - (37.30 \times 6.0) = -199.80$ kNm
$M_G = -(37.30 \times 3.0) = -111.90$ kNm



The values of the end-forces F1 to F12 can be determined by considering the equilibrium of each member and joint in turn.

Member Forces

Consider member ABC:

+ve ↑ $\Sigma F_y = 0$ $\quad +64.07 - F_1 = 0$ $\quad \therefore F_1 = 64.07$ kN ↓

+ve → $\Sigma F_x = 0$ $\quad +4.30 + 12.0 - F_2 = 0$ $\quad \therefore F_2 = 16.30$ kN ←

Consider Joint C:

+ve ↑ $\Sigma F_y = 0$ There is an applied vertical load at joint C = 15 kN ↓

$-F_1 + F_3 = -15.0$ $\quad \therefore F_3 = 49.07$ kN ↑

+ve → $\Sigma F_x = 0$ There is an applied horizontal load at joint C = 8 kN →

$-F_2 + F_4 = +8.0$ $\quad \therefore F_4 = 24.30$ kN →

Consider member CD:

+ve ↑ $\Sigma F_y = 0$ $\quad +49.07 - (12.0 \times 4.0) + F_5 = 0$ $\quad \therefore F_5 = -1.07$ kN ↓

+ve → $\Sigma F_x = 0$ $\quad +24.30 - F_6 = 0$ $\quad \therefore F_6 = 24.30$ kN ←

Consider member FGH:

+ve ↑ $\Sigma F_y = 0$ $\quad +78.93 - F_{11} = 0$ $\quad \therefore F_{11} = 78.93$ kN ↓

+ve → $\Sigma F_x = 0$ $\quad -37.30 + 8.0 + F_{12} = 0$ $\quad \therefore F_{12} = 29.30$ kN →

Consider Joint F:

+ve $\uparrow \Sigma F_y = 0$ There is an applied vertical load at joint F = 20 kN $\downarrow$

$F_{11} + F_9 = -20.0$ $\therefore F_9 = 58.93$ kN $\uparrow$

+ve $\rightarrow \Sigma F_x = 0$ There is an applied horizontal load at joint F = 5 kN $\rightarrow$

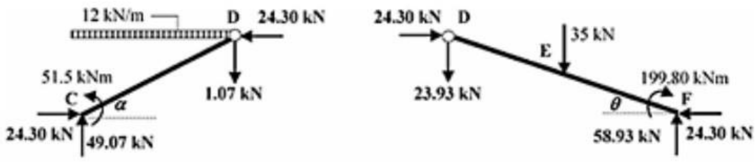
$+F_{12} - F_{10} = +5.0$ $\therefore F_{10} = 24.30$ kN $\leftarrow$

Consider member DF:

+ve $\uparrow \Sigma F_y = 0$ $\quad +58.93 - 35.0 + F_7 = 0$ $\quad \therefore F_7 = 23.93$ kN $\downarrow$

+ve $\rightarrow \Sigma F_x = 0$ $\quad -24.30 + F_8 = 0$ $\quad \therefore F_8 = 24.30$ kN $\rightarrow$

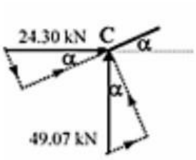
The calculated values can be checked by considering the equilibrium at joint D.



+ve $\rightarrow \Sigma F_x$ $\quad -24.30 + 24.30 = 0$

+ve $\uparrow \Sigma F_y$ $\quad -1.07 - 23.93 = -25.0$ kN (equal to the applied vertical load at D).

Member CD:



$\alpha = \tan^{-1}(2.0/4.0) = 26.565°$

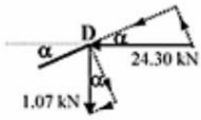
$\cos\alpha = 0.894$; $\quad \sin\alpha = 0.447$

Assume axial compression to be positive.

At joint C

Axial force = $+(24.30 \times \cos\alpha) + (49.07 \times \sin\alpha) = +43.66$ kN

Shear force = $-(24.30 \times \sin\alpha) + (49.07 \times \cos\alpha) = +33.01$ kN

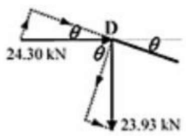


At joint D

Axial force = $+(24.30 \times \cos\alpha) + (1.07 \times \sin\alpha) = +22.20$ kN

Shear force = $-(24.30 \times \sin\alpha) + (49.07 \times \cos\alpha) = -9.91$ kN

Member DEF:



$\theta = \tan^{-1}(2.0/6.0) = 18.435°$

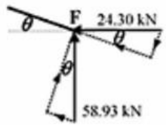
$\cos\theta = 0.947$; $\quad \sin\theta = 0.316$

Assume axial compression to be positive.

At joint D

Axial force = $+(24.30 \times \cos\theta) + (23.93 \times \sin\theta) = +30.57$ kN

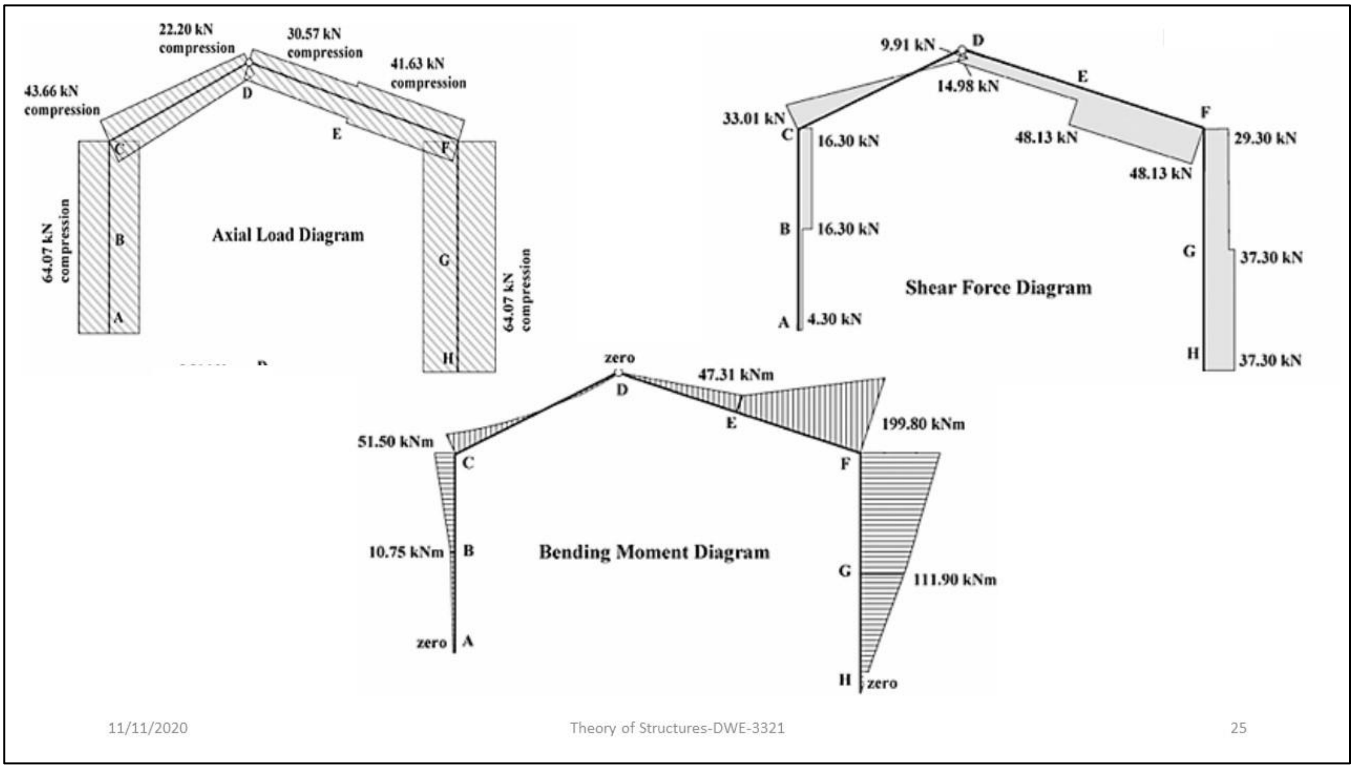
Shear force = $+(24.30 \times \sin\theta) - (23.93 \times \cos\theta) = +14.98$ kN



At joint F

Axial force = $+(24.30 \times \cos\theta) + (58.93 \times \sin\theta) = +41.63$ kN

Shear force = $-(24.30 \times \sin\theta) + (58.93 \times \cos\theta) = +48.13$ kN


22.20 kN
compression
30.57 kN
compression
41.63 kN
compression
43.66 kN
compression
64.07 kN
compression
64.07 kN
compression
Axial Load Diagram
9.91 kN
14.98 kN
33.01 kN
16.30 kN
48.13 kN
48.13 kN
29.30 kN
16.30 kN
37.30 kN
4.30 kN
37.30 kN
Shear Force Diagram
zero
47.31 kNm
199.80 kNm
51.50 kNm
10.75 kNm
Bending Moment Diagram
111.90 kNm
zero
zero
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UNIVERSITY OF ANBAR
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Unit-3

Analysis of Statically Determinate

Trusses

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Awesome Trusses

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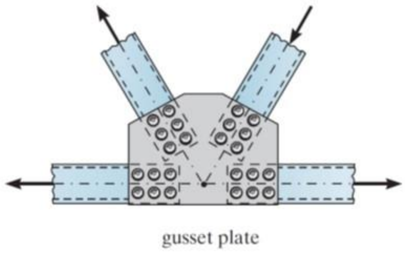
Common Types of Trusses:

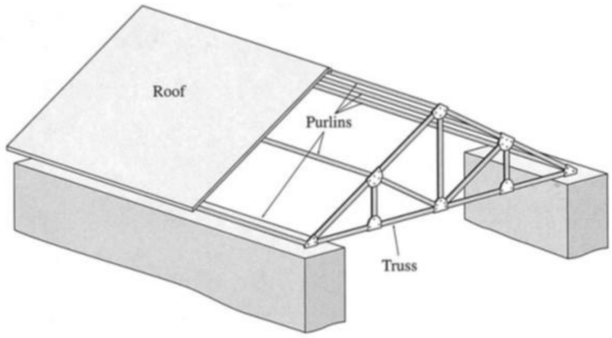
A truss is a structure composed of slender members joined together at their end points. The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a gusset plate, as shown in Fig. 3–1, or by simply passing a large bolt or pin through each of the members.



* Planar trusses lie in a single plane and are often used to support roofs and bridges.

Roof Trusses:

Roof trusses are often used as part of an industrial building frame, such as the one shown in Fig. 3–2 . Trusses used to support roofs are selected on the basis of the span, the slope, and the roof material.



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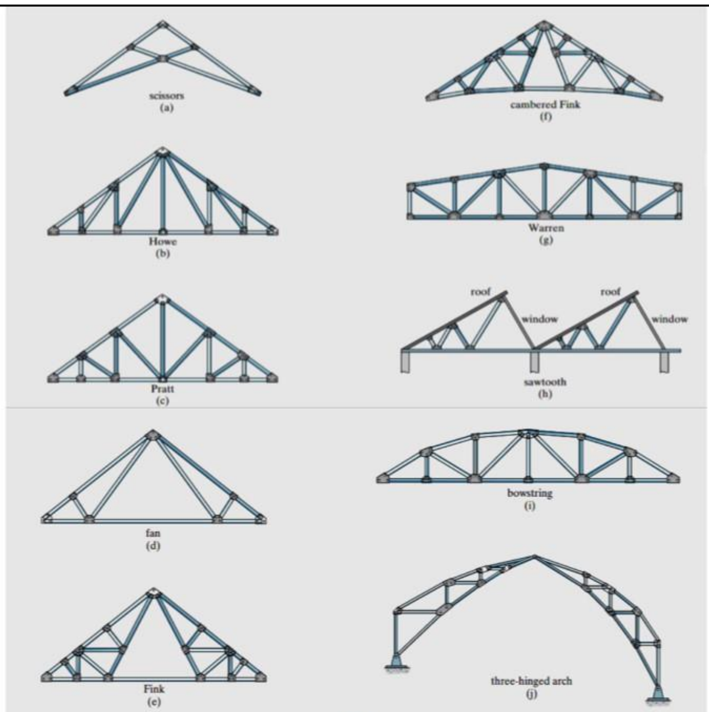

Fig. 3–3

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Types of Roof Trusses

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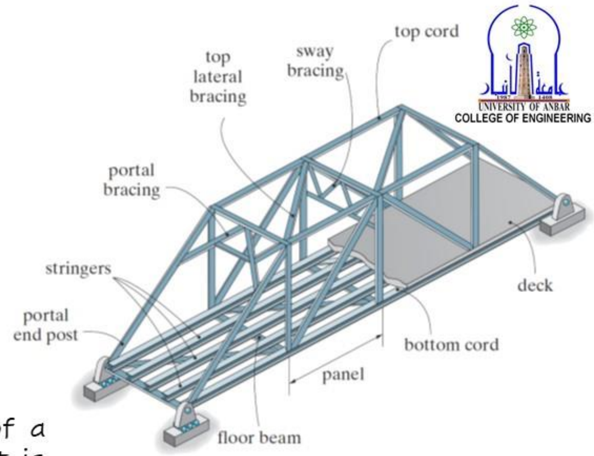
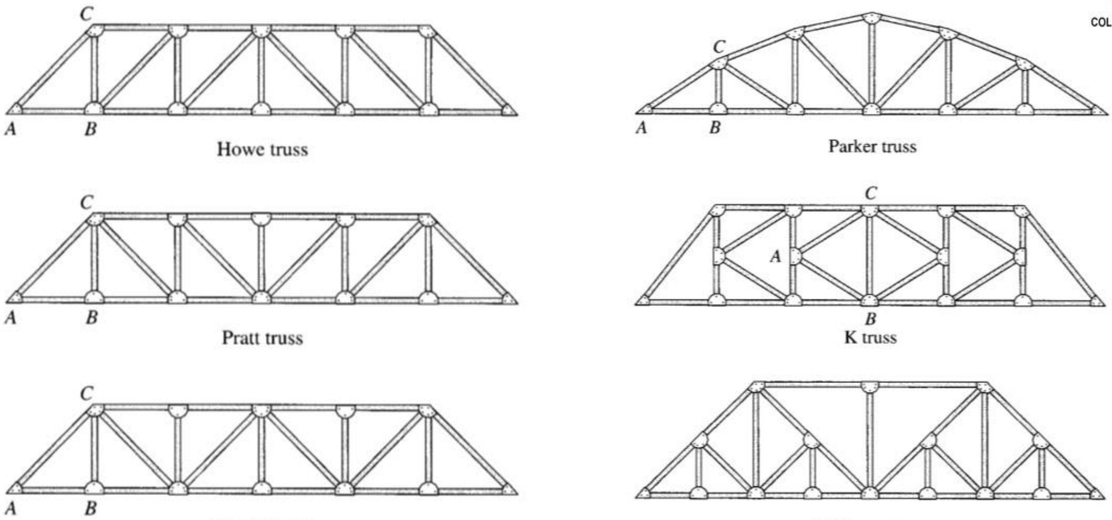


Bridge Trusses: The main structural elements of a typical bridge truss are shown in Fig. 3–4. Here it is seen that a load on the deck is first transmitted to stringers, then to floor beams, and finally to the joints of the two supporting side trusses. The top and bottom cords of these side trusses are connected by top and bottom lateral bracing, which serves to resist the lateral forces caused by wind and the sidesway caused by moving vehicles on the bridge.

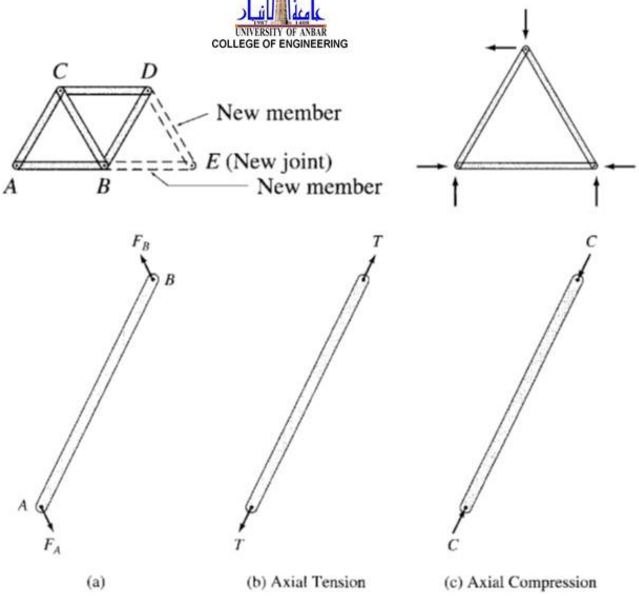
Common Types of Bridge Trusses



Howe truss | Parker truss | Pratt truss | K truss | Warren truss | Baltimore truss

Assumptions for Design: To design both the members and the connections of a truss, it is first necessary to determine the force developed in each member when the truss is subjected to a given loading. In this regard, two important assumptions will be made in order to idealize the truss.

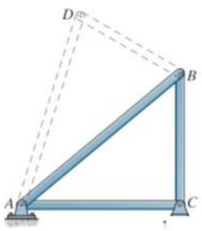
1. The members are joined together by smooth pins.
2. All loadings are applied at the joints.
3. Each truss member acts as an axial force member, and therefore the forces acting at the ends of the member must be directed along the axis of the member. If the force tends to elongate the member, it is a tensile force (T) ; whereas if the force tends to shorten the member, it is a compressive force (C).

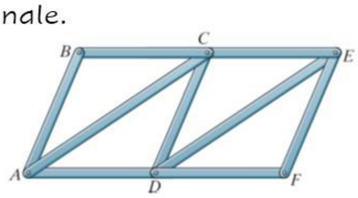


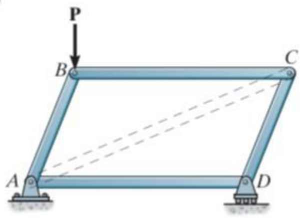
(a) (b) Axial Tension (c) Axial Compression

Classification of Coplanar Trusses:
Before beginning the force analysis of a truss, it is important to classify the truss as simple, compound, or complex, and then to be able to specify its determinacy and stability.

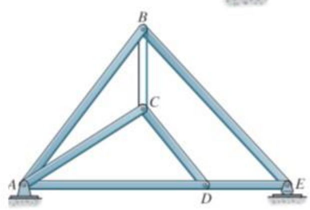
1) Simple Truss: The simplest framework that is rigid or stable is a trianale.







Therefore, a simple truss is constructed starting with a basic triangular element and connecting two members to form additional elements. As each additional element of two members is placed on a truss, the number of joints is increased by one.



2) Compound Truss :

This truss is formed by connecting two or more simple trusses together. This type of truss is often used for large spans.

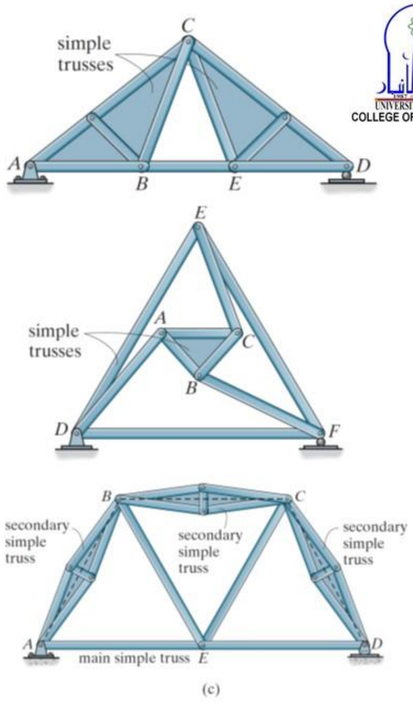
There are three ways in which simple trusses may be connected to form a compound truss:

A. Trusses may be connected by a common joint and bar.

B. Trusses may be joined by three bars.

C. Trusses may be joined where bars of a large simple truss, called the main truss, have been substituted by simple trusses, called secondary trusses.

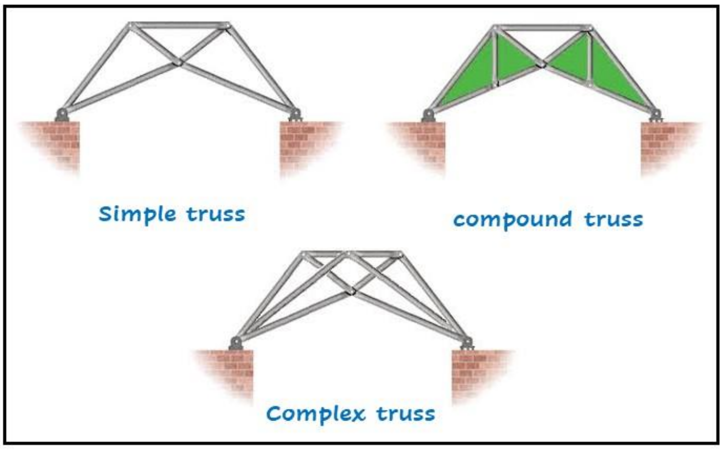
*Compound trusses are best analysed by applying both the method of joints and the method of sections.

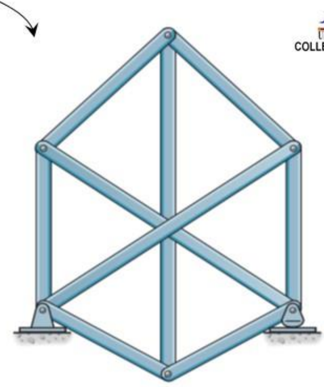


Various types of compound trusses

3) Complex Truss :

A complex truss is one that cannot be classified as being either simple or compound.

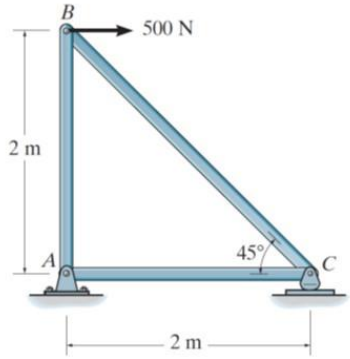




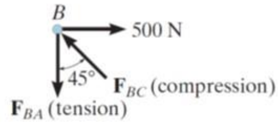
Complex truss

Method of Joints:

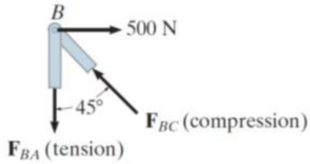
If a truss is in equilibrium, then each of its joints must also be in equilibrium. Hence, the method of joints consists of satisfying the equilibrium conditions $\sum F_x = 0$ and $\sum F_y = 0$ and for the forces exerted on the pin at each joint of the truss.



(a)



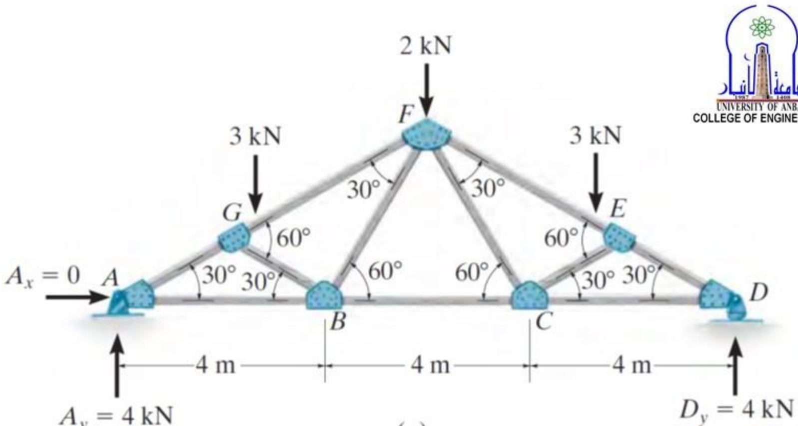
(b)



(c)

Example:

Determine the force in each member of the roof truss shown in the photo. The dimensions and loadings are shown in the figure. State whether the members are in tension or compression.



(a)

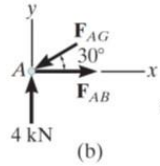
Solution:

Only the forces in half the members have to be determined, since the truss is symmetric with respect to both loading and geometry.

Joint A, We can start the analysis at joint A. **Why?**

$$+\uparrow\Sigma F_y = 0;\quad 4 - F_{AG}\sin 30^\circ = 0 \qquad F_{AG} = 8\text{ kN (C)}$$

$$\xrightarrow{+}\Sigma F_x = 0;\quad F_{AB} - 8\cos 30^\circ = 0 \qquad F_{AB} = 6.928\text{ kN (T)}$$



(b)

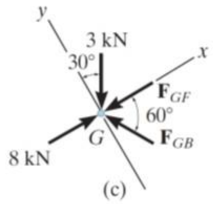
Joint G. In this case note how the orientation of the x, y axes avoids simultaneous solution of equations.

$$+\nwarrow\Sigma F_y = 0;\quad F_{GB}\sin 60^\circ - 3\cos 30^\circ = 0$$

$$F_{GB} = 3.00\text{ kN (C)}$$

$$+\nearrow\Sigma F_x = 0;\quad 8 - 3\sin 30^\circ - 3.00\cos 60^\circ - F_{GF} = 0$$

$$F_{GF} = 5.00\text{ kN (C)}$$



(c)

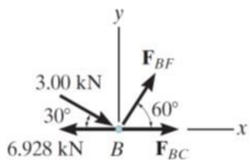
Joint B.

$$+\uparrow\Sigma F_y = 0;\quad F_{BF}\sin 60^\circ - 3.00\sin 30^\circ = 0$$

$$F_{BF} = 1.73\text{ kN (T)}$$

$$\xrightarrow{+}\Sigma F_x = 0;\quad F_{BC} + 1.73\cos 60^\circ + 3.00\cos 30^\circ - 6.928 = 0$$

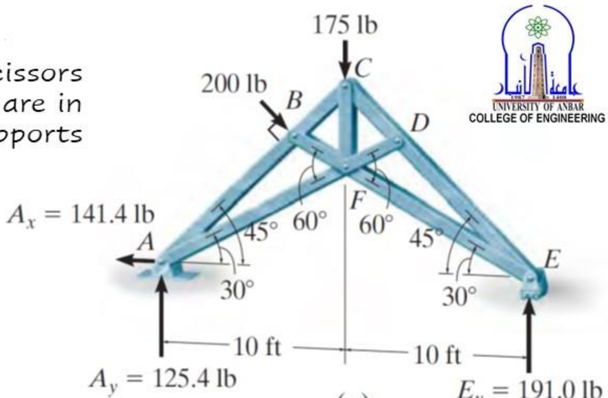
$$F_{BC} = 3.46\text{ kN (T)}$$



(d)

Example:

Determine the force in each member of the scissors truss shown figure. State whether the members are in tension or compression. The reactions at the supports are given.



(a)

Solution:

Joint E.

$$+\nearrow\Sigma F_y = 0;\qquad 191.0\cos 30^\circ - F_{ED}\sin 15^\circ = 0$$

$$F_{ED} = 639.1\text{ lb (C)}$$

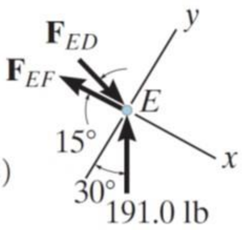
$$+\searrow\Sigma F_x = 0;\qquad 639.1\cos 15^\circ - F_{EF} - 191.0\sin 30^\circ = 0$$

$$F_{EF} = 521.8\text{ lb (T)}$$

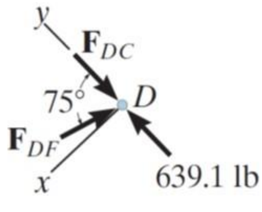
Joint D.

$$+\swarrow\Sigma F_x = 0;\qquad -F_{DF}\sin 75^\circ = 0 \qquad F_{DF} = 0$$

$$+\nwarrow\Sigma F_y = 0;\qquad -F_{DC} + 639.1 = 0 \qquad F_{DC} = 639.1\text{ lb (C)}$$



(b)

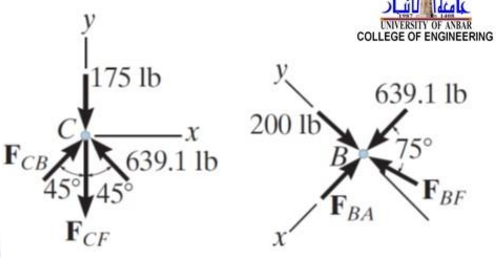


(c)

Joint C.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CB} \sin 45^\circ - 639.1 \sin 45^\circ = 0$$
$$F_{CB} = 639.1 \text{ lb (C)}$$

$$+\uparrow \Sigma F_y = 0; \qquad -F_{CF} - 175 + 2(639.1)\cos 45^\circ = 0$$
$$F_{CF} = 728.8 \text{ lb (T)}$$



Joint B.

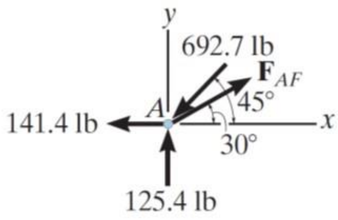
$$+\nwarrow \Sigma F_y = 0; \qquad F_{BF} \sin 75^\circ - 200 = 0 \quad F_{BF} = 207.1 \text{ lb (C)}$$

$$+\swarrow \Sigma F_x = 0; \qquad 639.1 + 207.1 \cos 75^\circ - F_{BA} = 0$$
$$F_{BA} = 692.7 \text{ lb (C)}$$

Joint A.

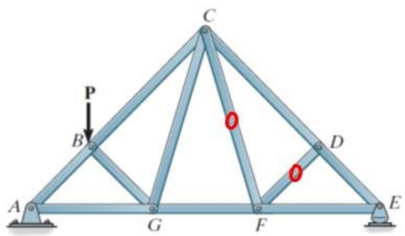
$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AF} \cos 30^\circ - 692.7 \cos 45^\circ - 141.4 = 0$$
$$F_{AF} = 728.9 \text{ lb (T)}$$

$$+\uparrow \Sigma F_y = 0; \qquad 125.4 - 692.7 \sin 45^\circ + 728.9 \sin 30^\circ = 0 \quad \textbf{Check}$$

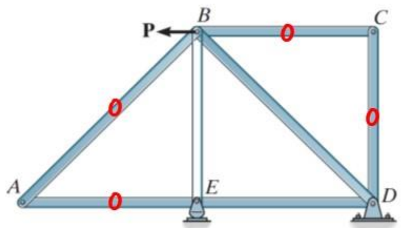


(f)

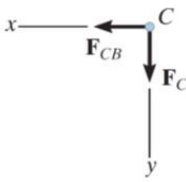
Zero – Force Members:



(a)



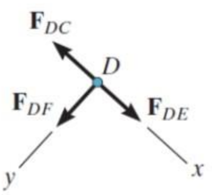
(a)



$\xleftarrow{+} \Sigma F_x = 0; F_{CB} = 0$

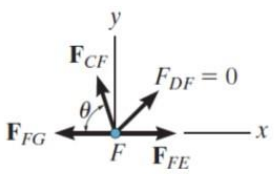
$+\downarrow \Sigma F_y = 0; F_{CD} = 0$

(b)



(b)

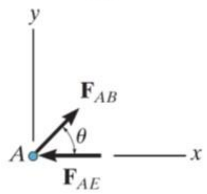
$+\swarrow \Sigma F_y = 0; F_{DF} = 0$



(c)

$+\uparrow \Sigma F_y = 0; F_{CF} \sin\theta + 0 = 0$

$F_{CF} = 0$ (since $\sin\theta \neq 0$)



$+\uparrow \Sigma F_y = 0; F_{AB} \sin\theta = 0$

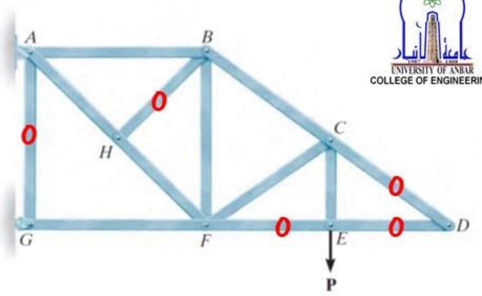
$F_{AB} = 0$ (since $\sin\theta \neq 0$)

$\xrightarrow{+} \Sigma F_x = 0; -F_{AE} + 0 = 0$

$F_{AE} = 0$

(c)

Example:
Using the method of joints, indicate all the members of the truss shown in figure that have zero force.

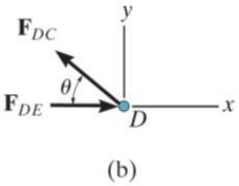


Solution:

Joint D.

$$+\uparrow \Sigma F_y = 0; \qquad F_{DC} \sin\theta = 0 \qquad F_{DC} = 0$$

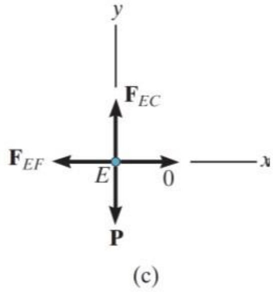
$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{DE} + 0 = 0 \qquad F_{DE} = 0$$



(b)

Joint E.

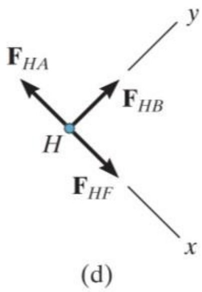
$$\xleftarrow{+} \Sigma F_x = 0; \qquad F_{EF} = 0$$



(c)

Joint H.

$$+\nearrow \Sigma F_y = 0; \qquad F_{HB} = 0$$



(d)

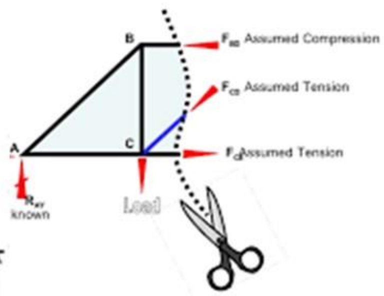
Joint G.

$$+\uparrow \Sigma F_y = 0; \qquad F_{GA} = 0$$

Method of Sections:

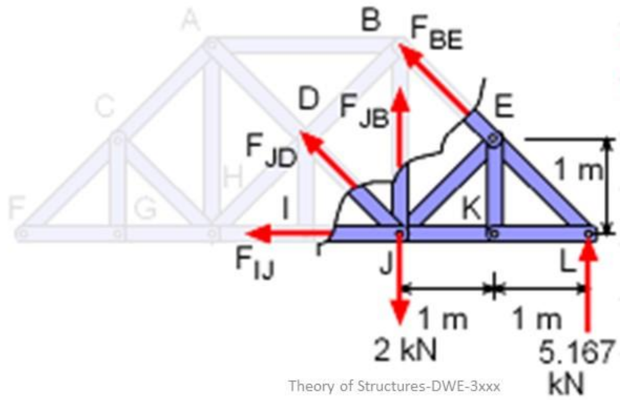
When the method of sections is used to determine the force in a particular member, a decision must be made as to how to "cut" or section the truss. Since only three independent equilibrium equations ($\sum F_x = 0$, $\sum F_y = 0$ **and** $\sum M_o = 0$) can be applied to the isolated portion of the truss, try to select a section that, in general, passes through not more than three members in which the forces are unknown.

Assume the forces on cut members act as external forces on the cut

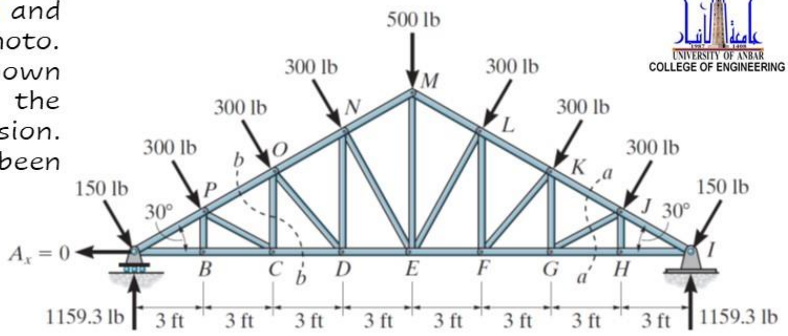


$$\sum M_L = 0$$

$$\sum M_J = 0$$



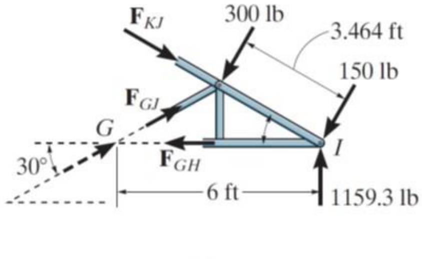
Example:
Determine the force in members **GJ** and **CO** of the roof truss shown in the photo. The dimensions and loadings are shown in the figure. State whether the members are in tension or compression. The reactions at the supports have been calculated.



(a)

Solution:

Member GJ.
Free-Body Diagram. The force in member GJ can be obtained by considering the section aa . Taking the free-body diagram of the right part of this section:

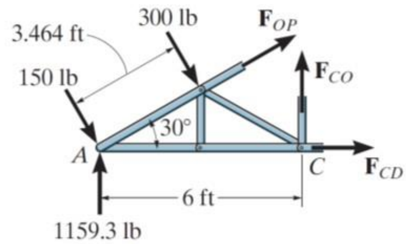


(b)

$$\circlearrowright + \Sigma M_I = 0; \qquad -F_{GJ} \sin 30°(6) + 300(3.464) = 0$$

$$F_{GJ} = 346 \text{ lb (C)}$$

Member CO.
The force in CO can be obtained by using section bb. Taking the free-body diagram of the left portion of the section:



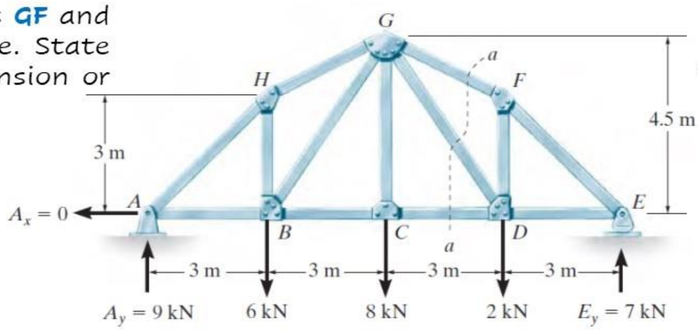
(c)

$$\circlearrowright + \Sigma M_A = 0; \qquad -300(3.464) + F_{CO}(6) = 0$$

$$F_{CO} = 173 \text{ lb (T)}$$

Example:

Determine the force in members **GF** and **GD** of the truss shown in figure. State whether the members are in tension or compression.

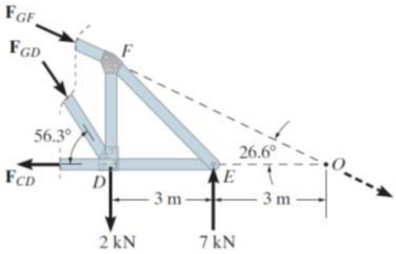


(a)

Solution:

$$\circlearrowright\!+\Sigma M_D = 0; \qquad -F_{GF}\sin 26.6°(6) + 7(3) = 0$$
$$F_{GF} = 7.83 \text{ kN (C)}$$

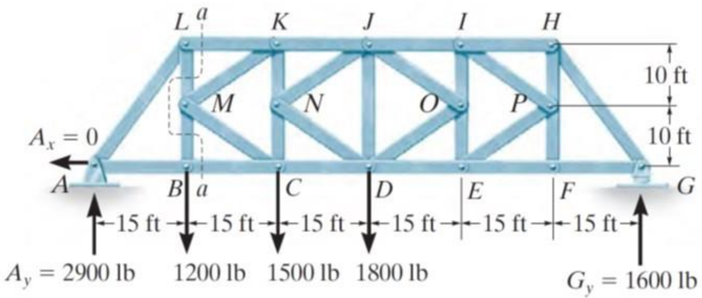
$$\circlearrowright\!+\Sigma M_O = 0; \qquad -7(3) + 2(6) + F_{GD}\sin 56.3°(6) = 0$$
$$F_{GD} = 1.80 \text{ kN (C)}$$



(b)

Example:

Determine the force in members **BC** and **MC** of the K-truss shown in the figure. State whether the members are in tension or compression.



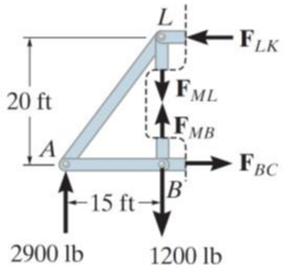
(a)

Solution:

$$\circlearrowright\!+\Sigma M_L = 0; \qquad -2900(15) + F_{BC}(20) = 0$$
$$F_{BC} = 2175 \text{ lb (T)}$$

The force in **MC** can be obtained indirectly by first obtaining the force in **MB** from vertical force equilibrium of joint B, i.e., **F_{MB}=1200 lb (T)** Then:

$$+\uparrow\Sigma F_y = 0; \qquad 2900 - 1200 + 1200 - F_{ML} = 0$$
$$F_{ML} = 2900 \text{ lb (T)}$$

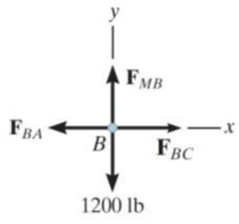


(b)

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \left(\frac{3}{\sqrt{13}}\right)F_{MC} - \left(\frac{3}{\sqrt{13}}\right)F_{MK} = 0$$

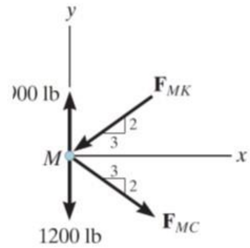
$$+\uparrow \Sigma F_y = 0; \qquad 2900 - 1200 - \left(\frac{2}{\sqrt{13}}\right)F_{MC} - \left(\frac{2}{\sqrt{13}}\right)F_{MK} = 0$$

$$F_{MK} = 1532 \text{ lb (C)} \qquad F_{MC} = 1532 \text{ lb (T)} \qquad \textit{Ans.}$$



(c)

Hint:
It is also possible to solve for the force in **MC** by using the result for In this case, pass a vertical section through **LK,MK,MC**, and **BC**. Isolate the left section and apply $\sum M_K = 0$.



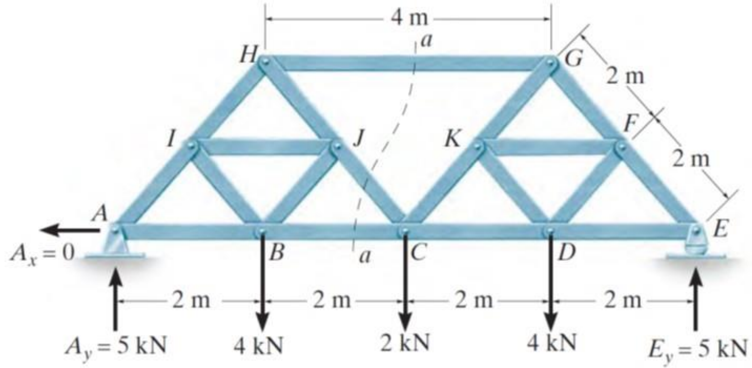
(d)

Compound Trusses:

If this type of truss is best analysed by applying both the method of joints and the method of sections. It is often convenient to first recognize the type of construction and then perform the analysis using the following procedure.

Mixed Analysis Method:

Compound trusses can be analysed using mixed method where section method can be used to find member forces that will help in solving the other ones using joint method or vice versa.

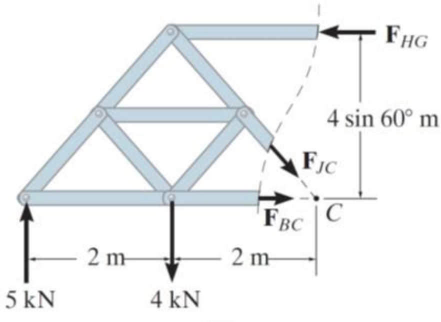


(a)

Solution:

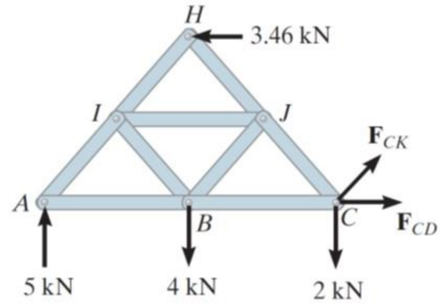
$$\circlearrowleft + \Sigma M_C = 0; \quad -5(4) + 4(2) + F_{HG}(4 \sin 60°) = 0$$

$$F_{HG} = 3.46 \text{ kN (C)}$$



(b)

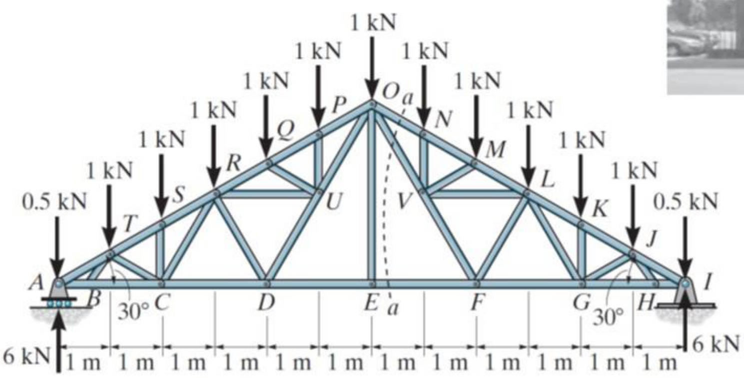
Joint A: Determine the force in *AB* and *AI*.
Joint H: Determine the force in *HI* and *HJ*.
Joint I: Determine the force in *IJ* and *IB*.
Joint B: Determine the force in *BC* and *BJ*.
Joint J: Determine the force in *JC*.



(c)

Example:

Compound roof trusses are used in a garden centre, as shown in the photo. They have the dimensions and loading shown in Fig. a. Indicate how to analyse this truss.



(a)

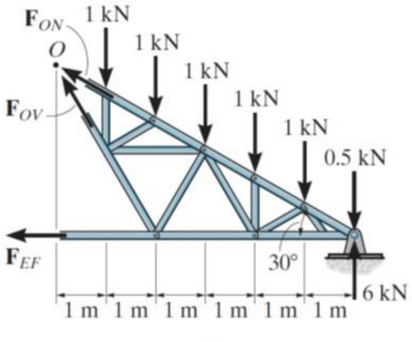
Solution:

$$\circlearrowleft + \Sigma M_O = 0; \quad -1(1) - 1(2) - 1(3) - 1(4) - 1(5) - 0.5(6) + 6(6) - F_{EF}(6 \tan 30°) = 0$$

$$F_{EF} = 5.20 \text{ kN (T)} \qquad \textit{Ans.}$$

By inspection notice that BT, EO, and HJ are zero-force members since $+\uparrow \Sigma F_y = 0$ at joints B, E, and H, respectively. Also, by applying $+\nwarrow \Sigma F_y = 0$ (perpendicular to AO) at joints P, Q, S, and T, we can directly determine the force in members PU, QU, SC, and TC, respectively.

It is a good practice to try solving it yourself !



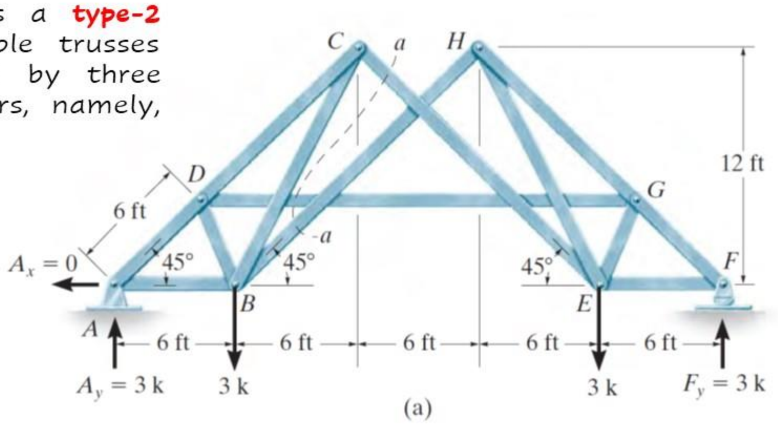
(b)

Example:

Indicate how to analyse the compound truss shown in the figure.

Solution:

The truss may be classified as a **type-2** compound truss since the simple trusses **ABCD** and **FEHG** are connected by three nonparallel or nonconcurrent bars, namely, **CE**, **BH**, and **DG**.



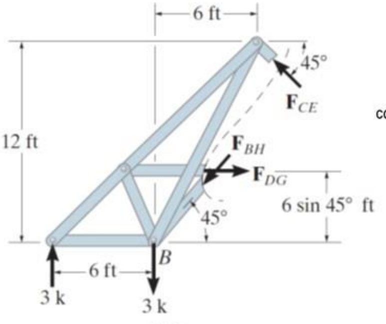
(a)

$$\circlearrowleft + \Sigma M_B = 0; \quad -3(6) - F_{DG}(6 \sin 45^\circ) + F_{CE} \cos 45^\circ (12) + F_{CE} \sin 45^\circ (6) = 0$$

$$+\uparrow \Sigma F_y = 0; \quad 3 - 3 - F_{BH} \sin 45^\circ + F_{CE} \sin 45^\circ = 0$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{BH} \cos 45^\circ + F_{DG} - F_{CE} \cos 45^\circ = 0$$

$$F_{BH} = F_{CE} = 2.68 \text{ k (C)} \qquad F_{DG} = 3.78 \text{ k (T)}$$

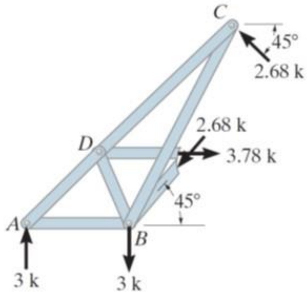


(b)

Practice, Practice, and Practice !

Hint:

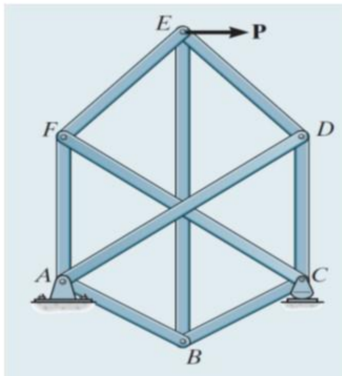
Joint A: Determine the force in *AB* and *AD*.
Joint D: Determine the force in *DC* and *DB*.
Joint C: Determine the force in *CB*.



(c)

Complex Trusses:

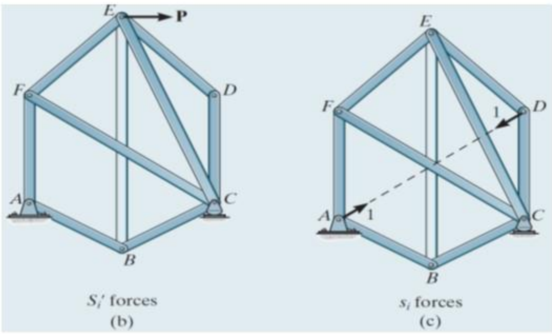
If this The member forces in a complex truss can be determined using the method of joints; however, the solution will require writing the two equilibrium equations for each of the j joints of the truss and then solving the complete set of 2j equations simultaneously. This approach may be impractical for hand calculations, especially in the case of large trusses. Therefore, a more direct method for analysing a complex truss, referred to as the method of substitute members, will be presented here.



(a)

Procedure of Analysis:

1- Reduction to Stable Simple Truss
2- External Loading on Simple Truss
3- Remove External Loading from Simple Truss
4- Superposition



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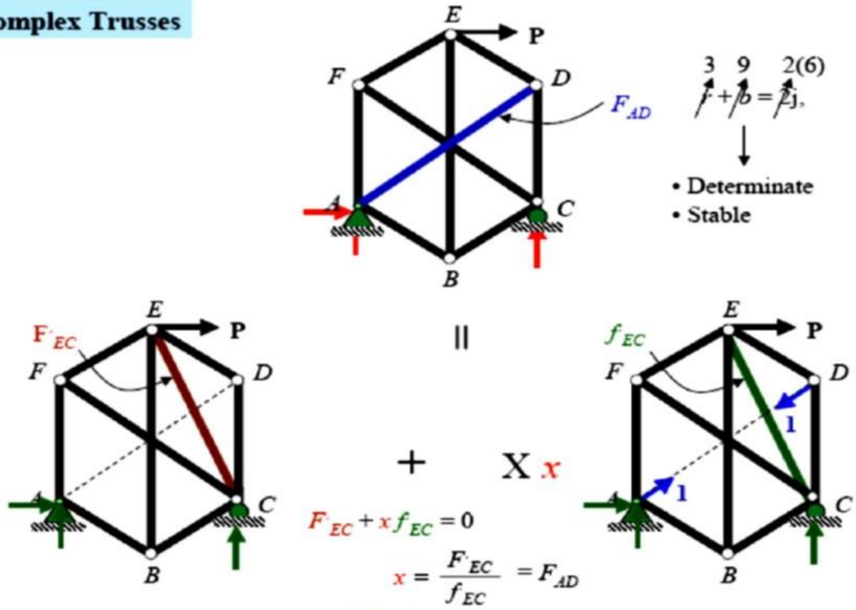
Complex Trusses



$F'_{EC} + x f_{EC} = 0$

$x = \frac{F'_{EC}}{f_{EC}} = F_{AD}$

$F_i = F'_i + x f_i$

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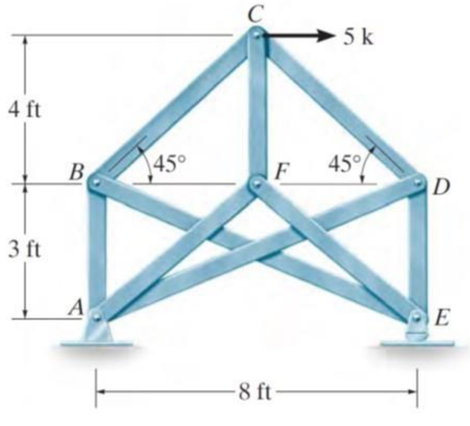
Example:

Determine the force in each member of the complex truss shown in the figure. Assume joints B, F, and D are on the same horizontal line.

State whether the members are in tension or compression.

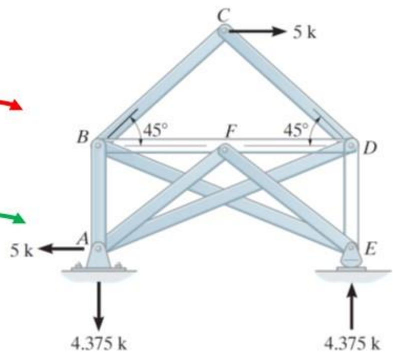
Solution:



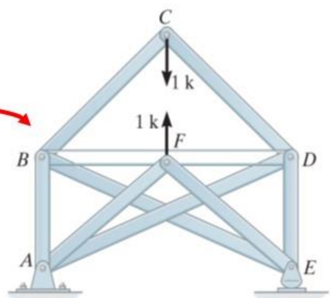
(a)

1- Reduction to Stable Simple Truss

2- External Loading on Simple Truss



3- Remove External Loading from Simple Truss



(c)

4- Superposition

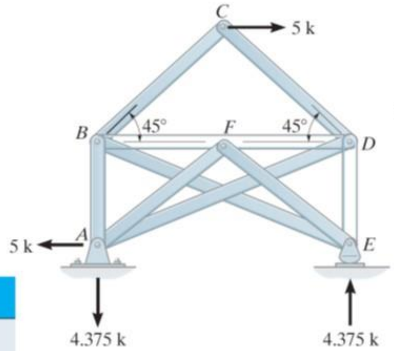
$$S_{DB} = S'_{DB} + xs_{DB} = 0$$

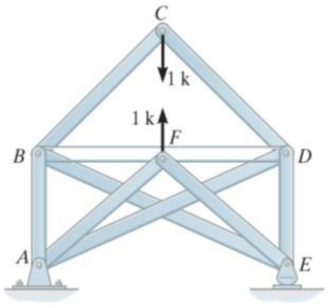
$$-2.50 + x(1.167) = 0 \qquad x = 2.143$$

$$S_i = S'_i + xs_i$$

TABLE 1

Member	S'_i	s_i	xs_i	S_i
CB	3.54	−0.707	−1.52	2.02 (T)
CD	−3.54	−0.707	−1.52	5.05 (C)
FA	0	0.833	1.79	1.79 (T)
FE	0	0.833	1.79	1.79 (T)
EB	0	−0.712	−1.53	1.53 (C)
ED	−4.38	−0.250	−0.536	4.91 (C)
DA	5.34	−0.712	−1.53	3.81 (T)
DB	−2.50	1.167	2.50	0
BA	2.50	−0.250	−0.536	1.96 (T)
CB				2.14 (T)





(c)

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Unit-4

Approximate Analysis of Structures

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Awesome Structures

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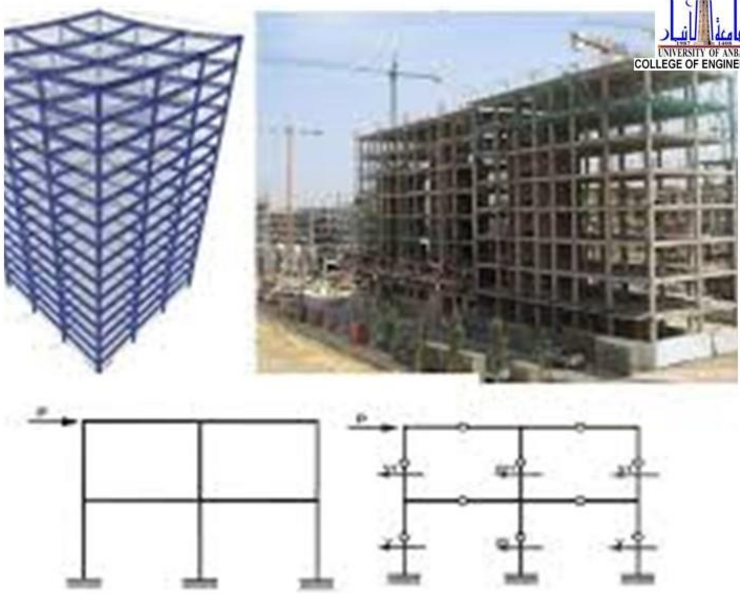
Approximate Analysis of Tall Buildings under Lateral Loadings:

1. Portal Frame Method
2. Cantilever Beam Method

Causes of Lateral Loading:

1. Wind
2. Earthquakes



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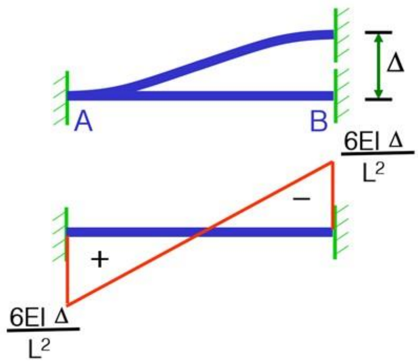
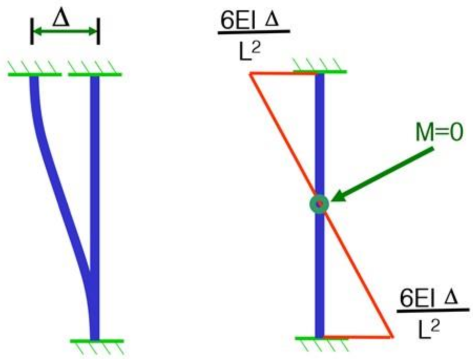
BEAMS ⇔ COLUMNS





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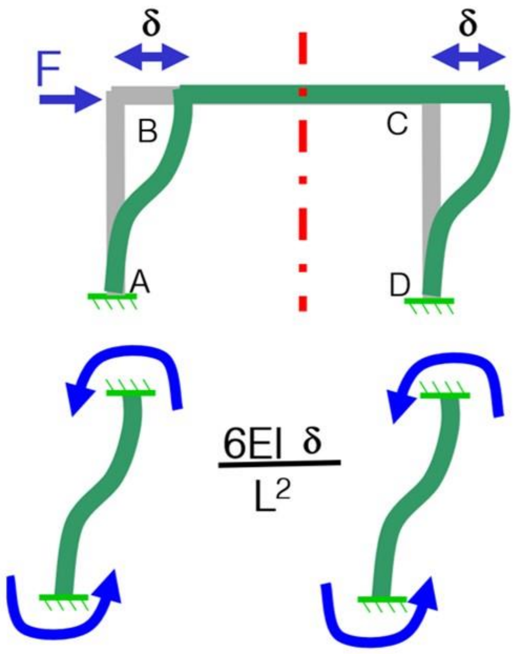
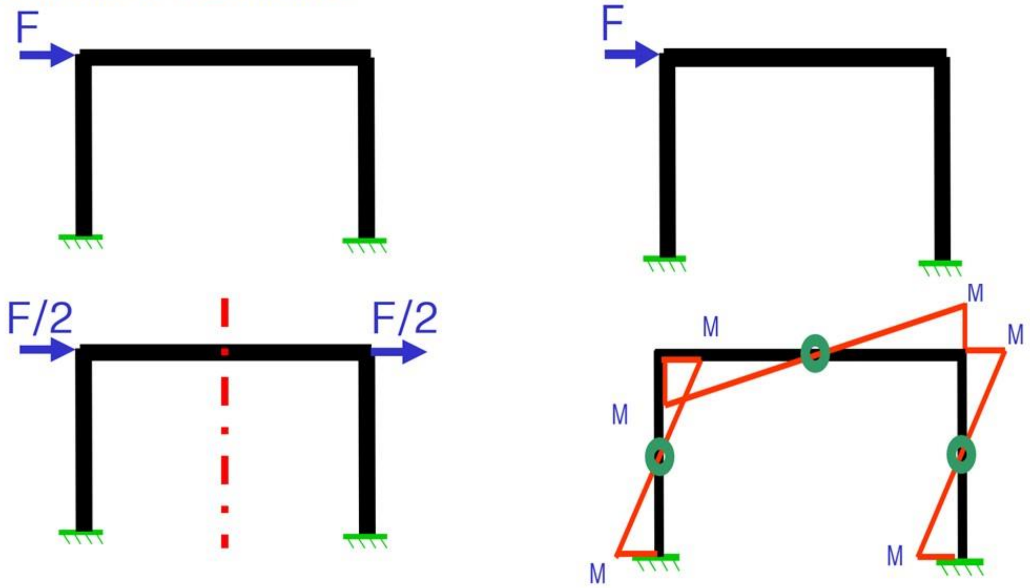
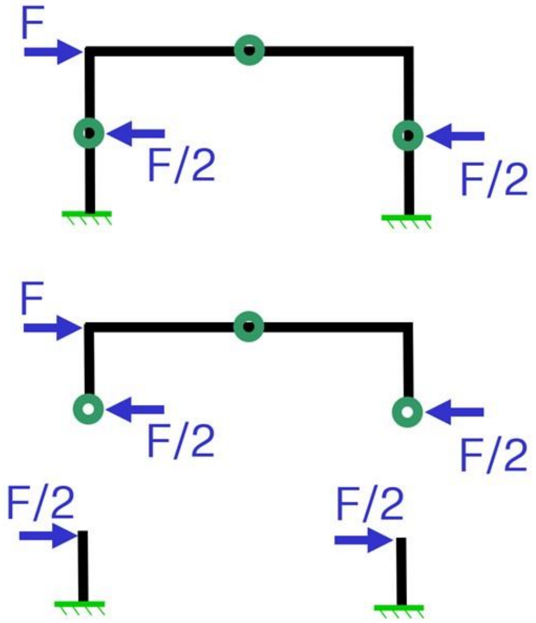
Anti-Symmetry



Criterion-1: when frame is subjected to lateral loads, we can put intermediate hinge in the mid of each member



One Storey Frames:



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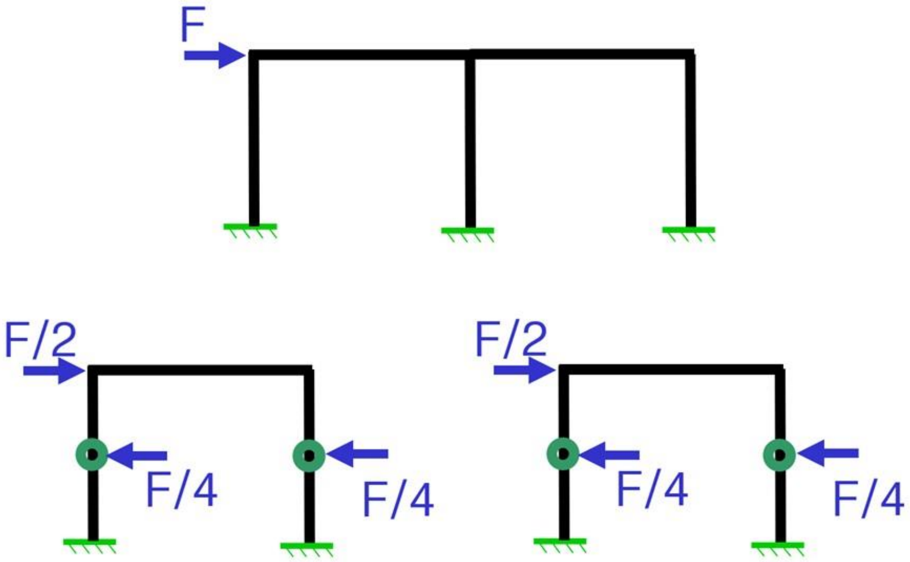

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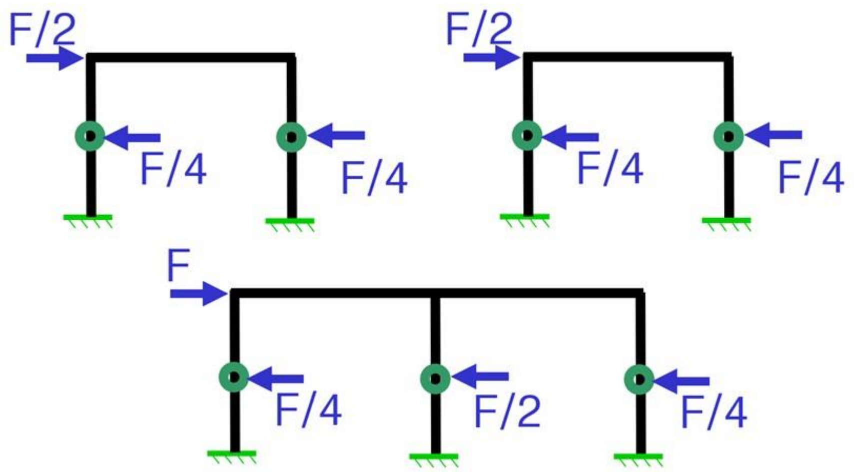
Criterion-2: When frame is subjected to lateral loads, the interior column carries the double of the exterior columns



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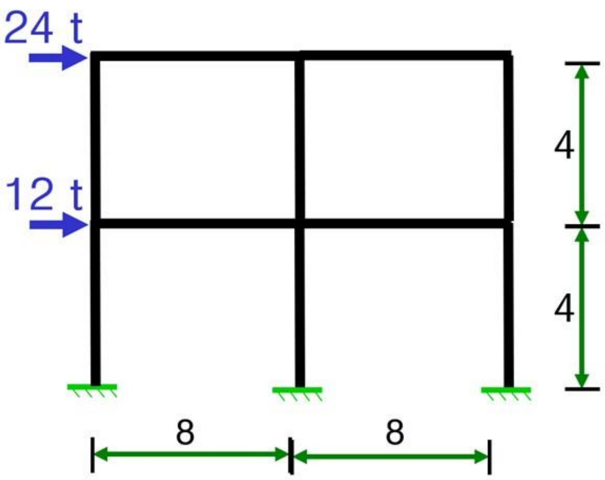
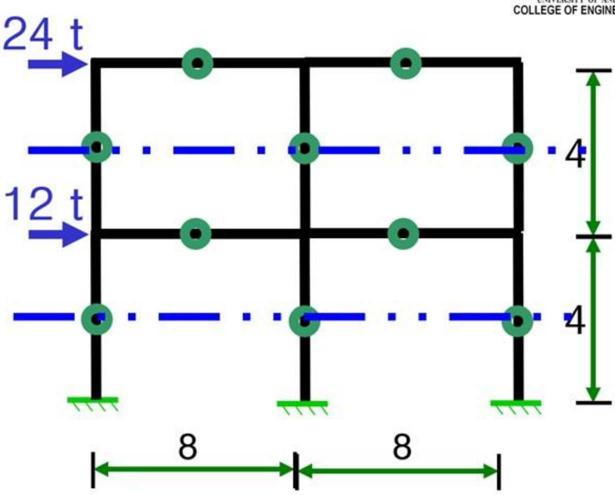
Example-1:





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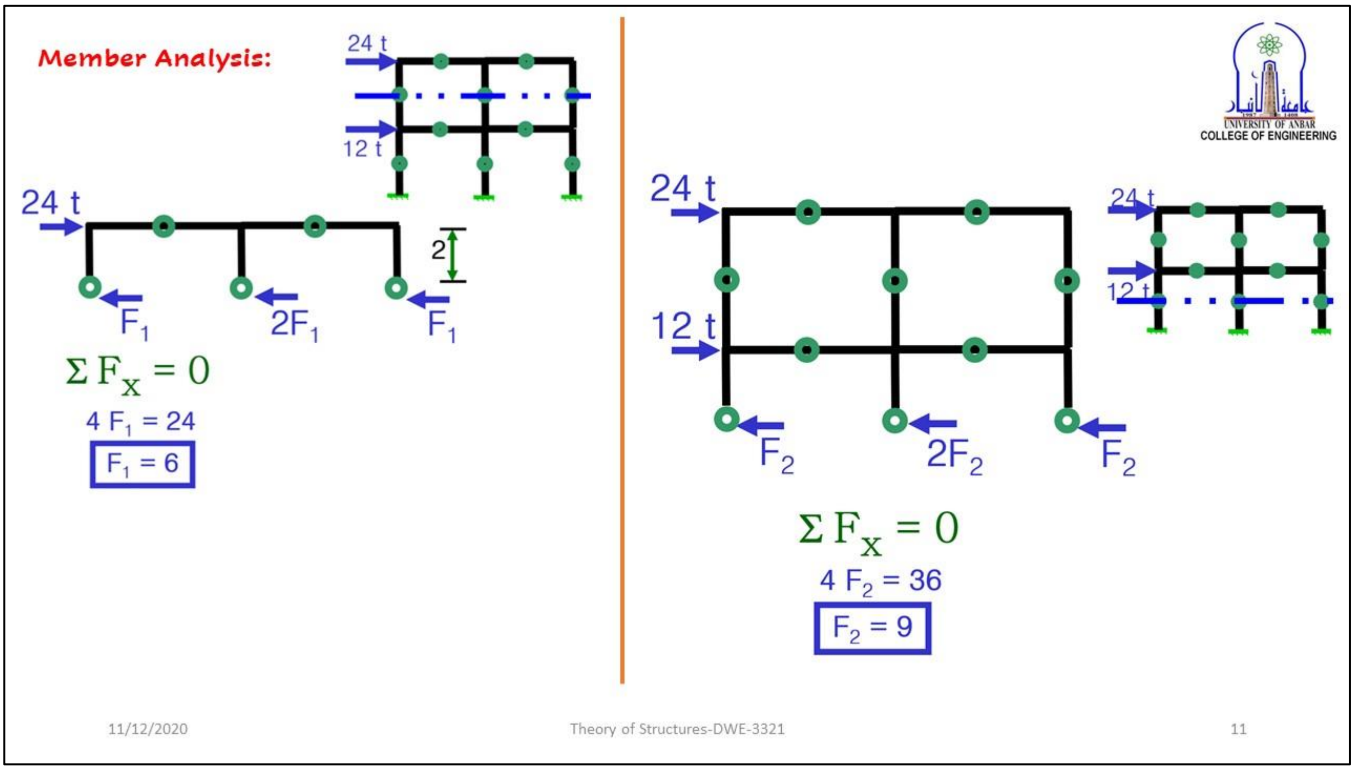
Member Analysis:
24 t
12 t
24 t
2
F1
2F1
F1
ΣFx = 0
4 F1 = 24
F1 = 6
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24 t
12 t
F2
2F2
F2
24 t
12 t
ΣFx = 0
4 F2 = 36
F2 = 9
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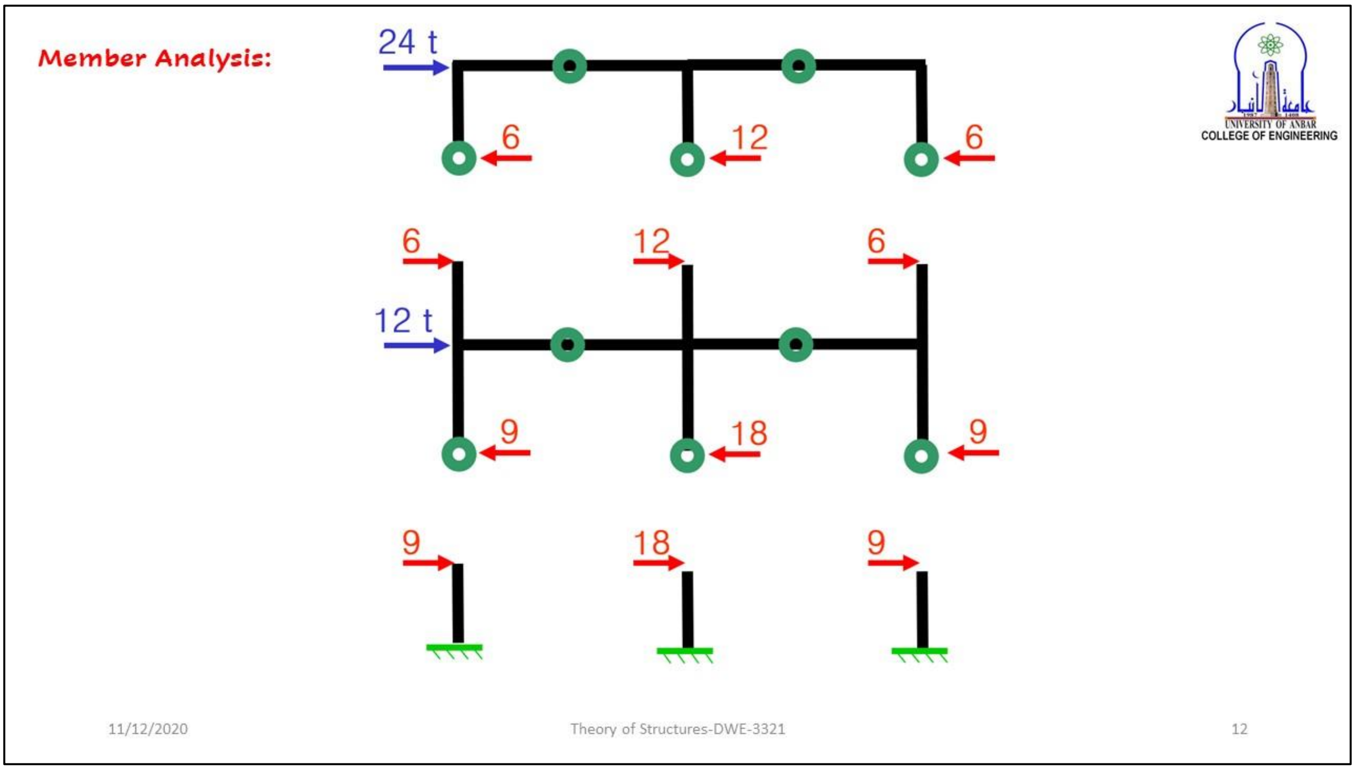
Member Analysis:
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24 t
6
12
6
6
12
6
12 t
9
18
9
9
18
9
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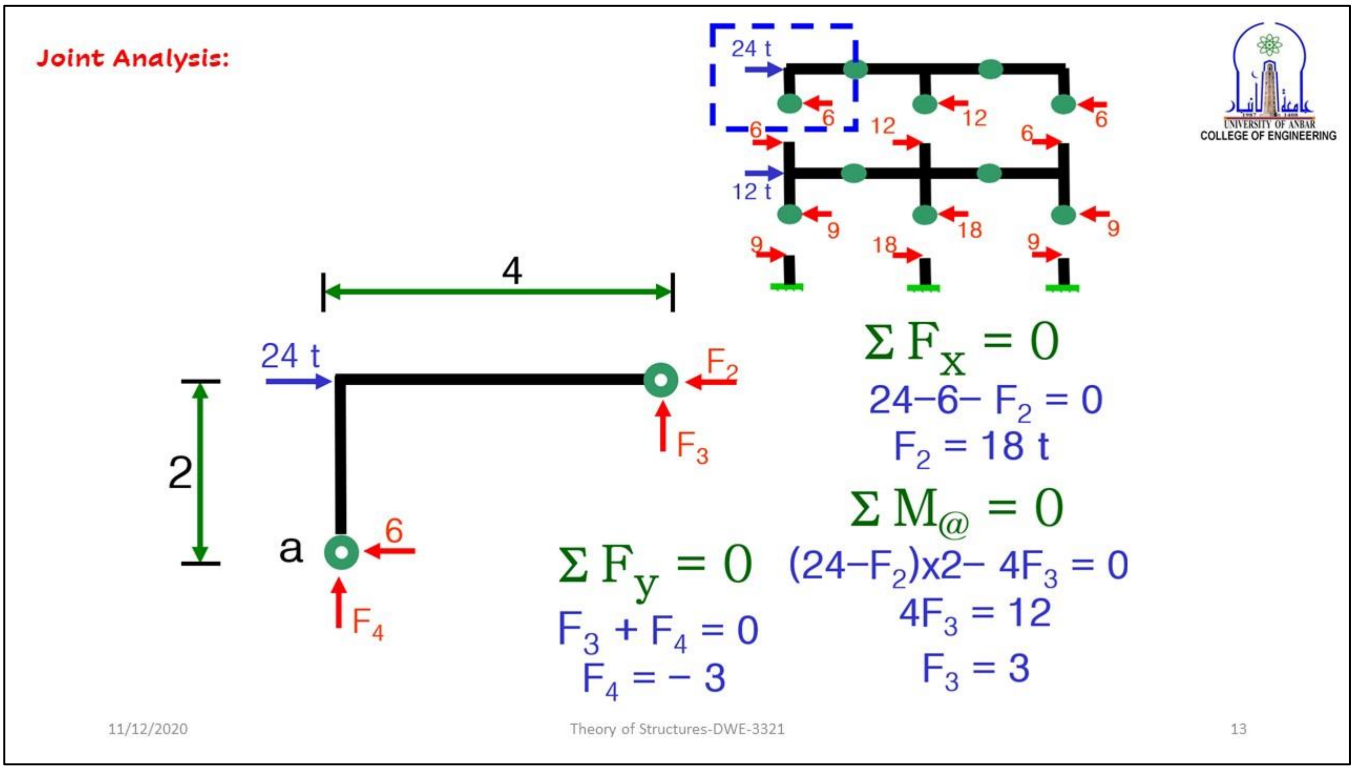
12


Joint Analysis:
24 t
12 t
4
2
a
6
F2
F3
F4
ΣFx = 0
24−6− F2 = 0
F2 = 18 t
ΣM@ = 0
(24−F2)x2− 4F3 = 0
4F3 = 12
F3 = 3
ΣFy = 0
F3 + F4 = 0
F4 = − 3
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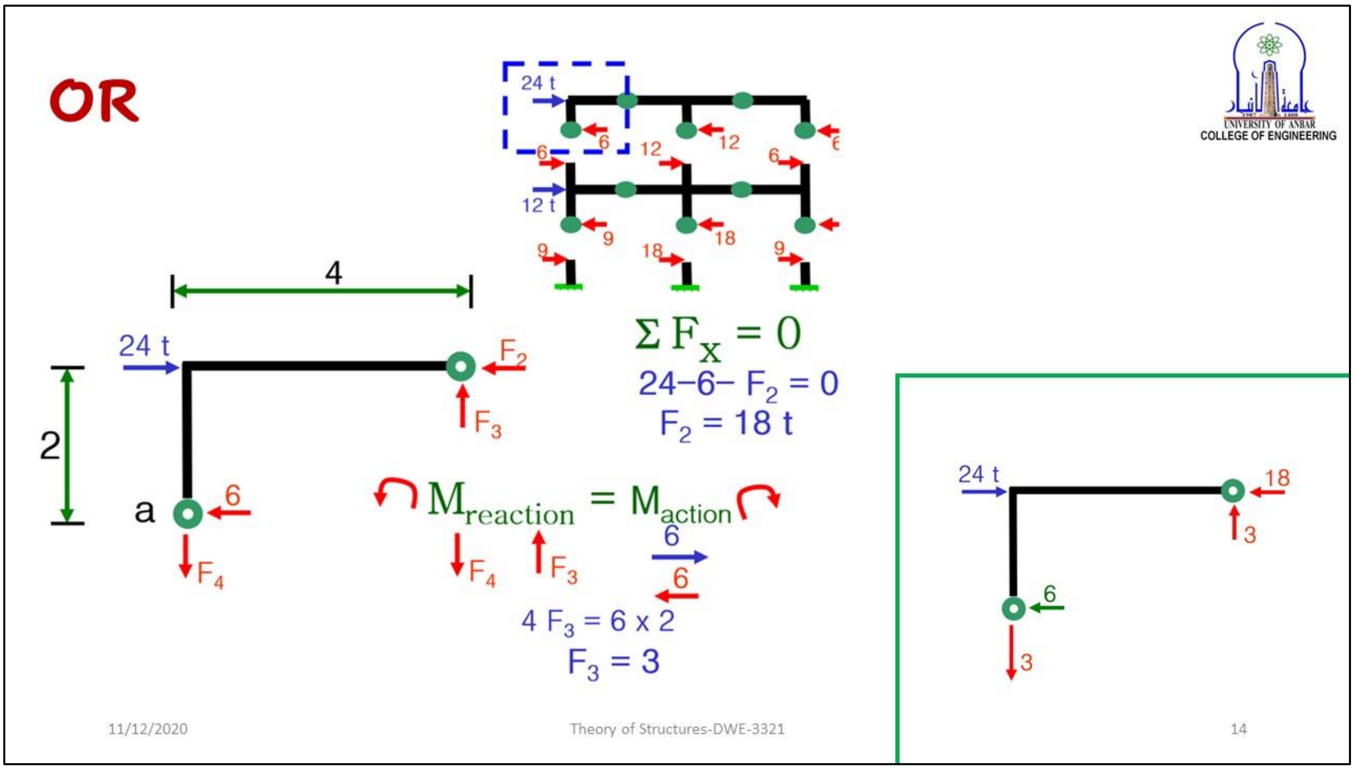
OR
24 t
12 t
4
2
a
6
F2
F3
F4
ΣFx = 0
24−6− F2 = 0
F2 = 18 t
Mreaction = Maction
4 F3 = 6 x 2
F3 = 3
24 t
18
3
6
3
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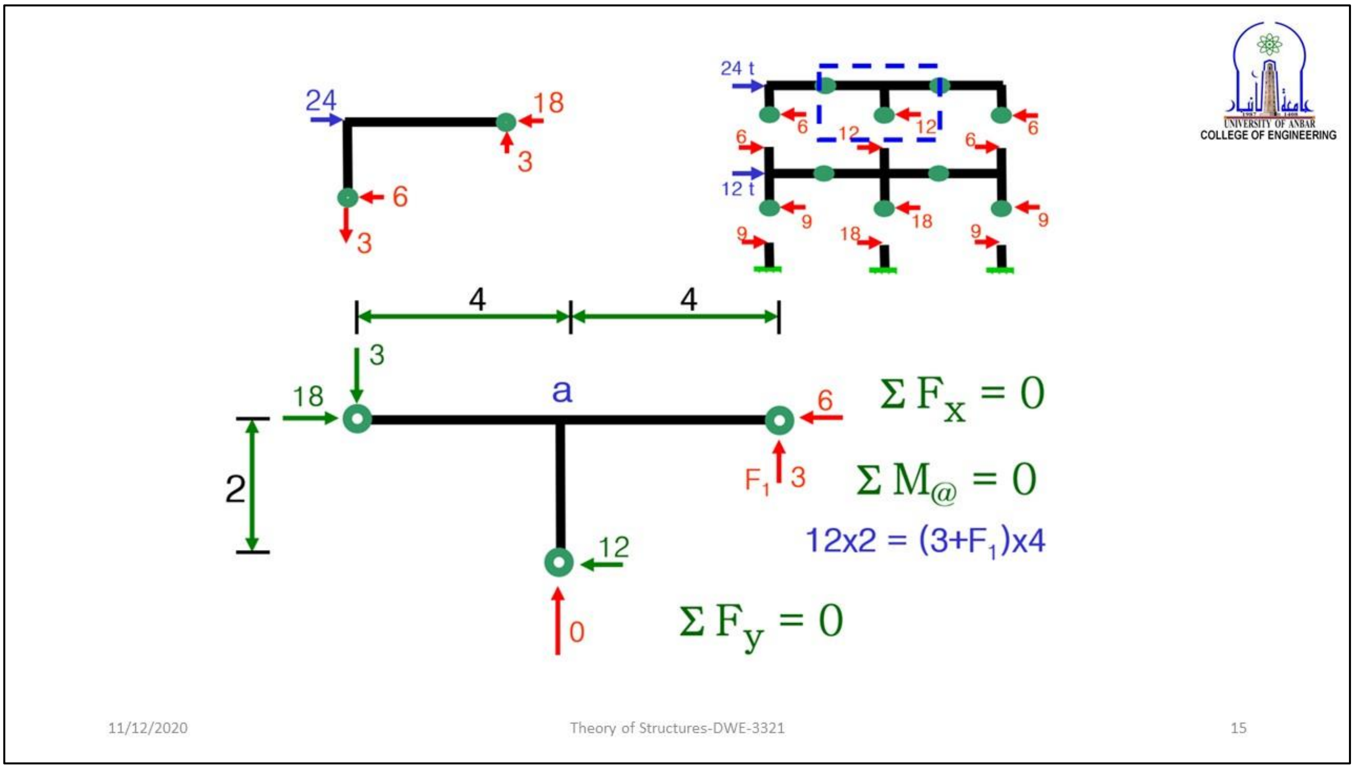
24
18
3
6
3
24 t
6
12
12
6
6
6
12 t
9
18
9
9
18
9
4
4
3
18
a
6
2
12
0
F1
3
ΣFx = 0
ΣM@ = 0
12x2 = (3+F1)x4
ΣFy = 0
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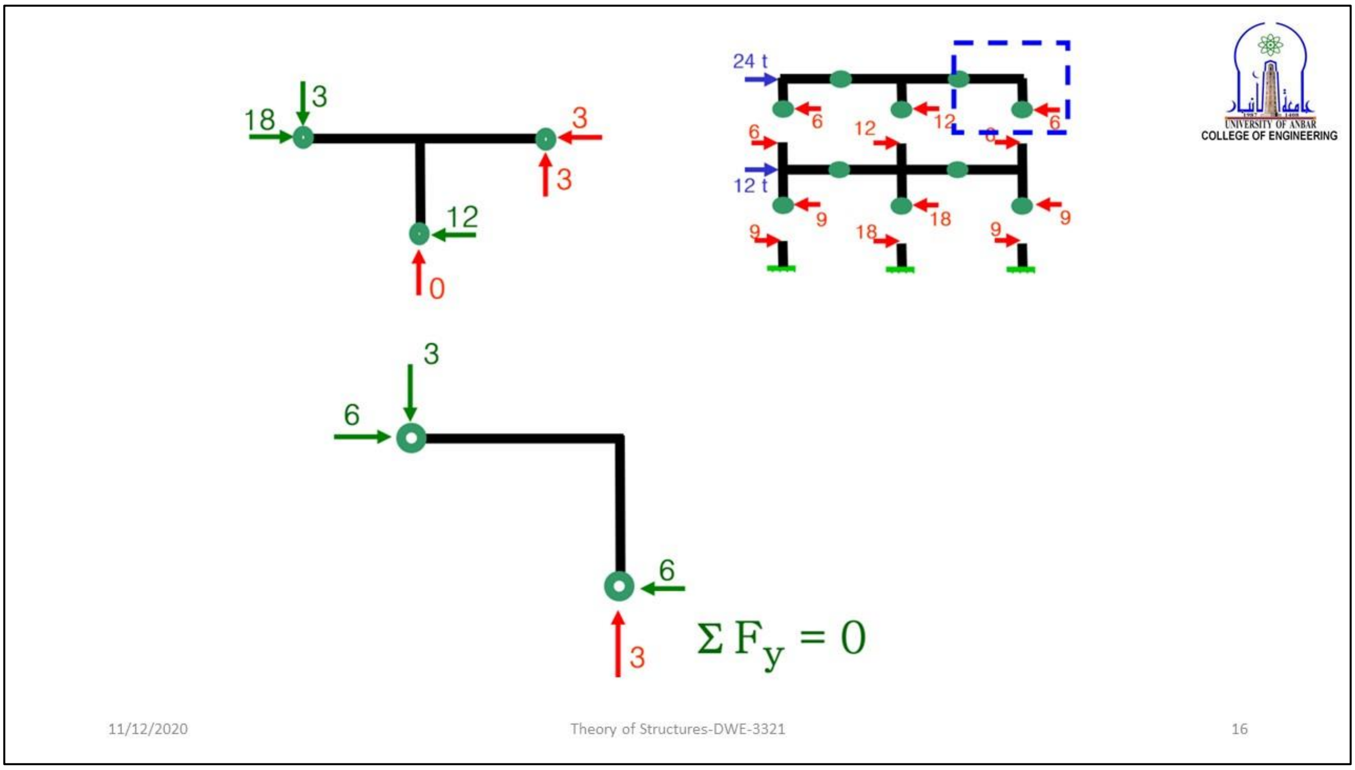
18
3
3
3
12
0
24 t
6
12
6
6
12
12 t
9
18
9
9
18
9
3
6
6
3
ΣFy = 0
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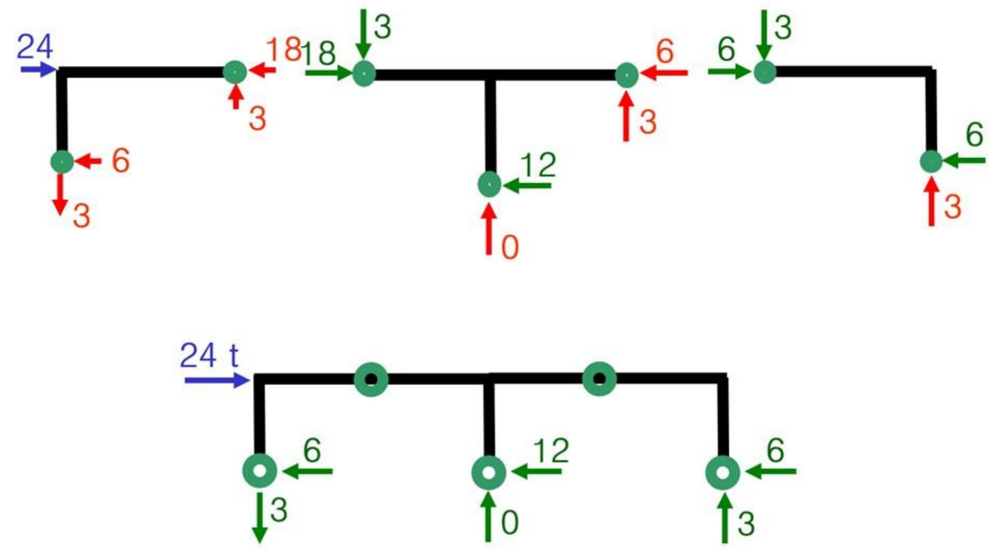

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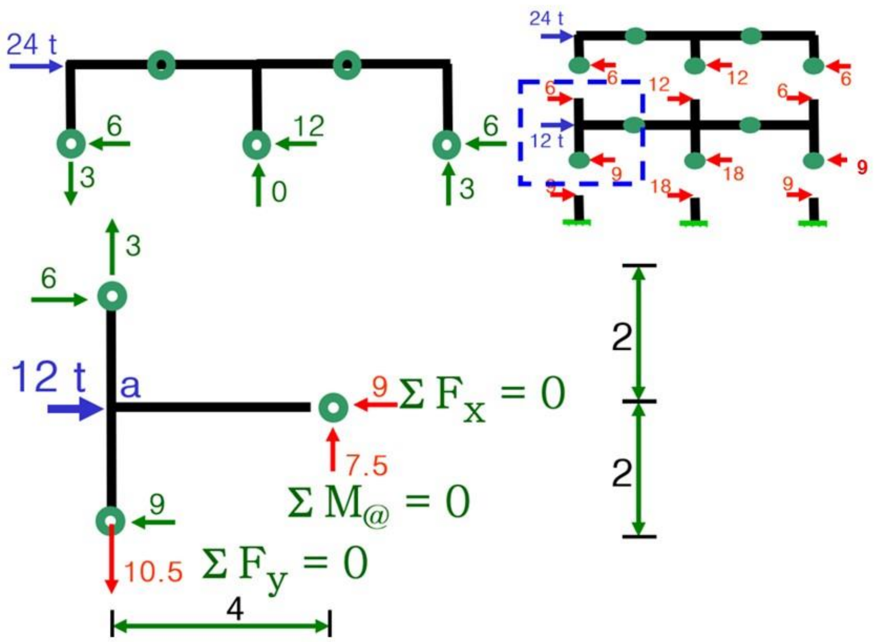

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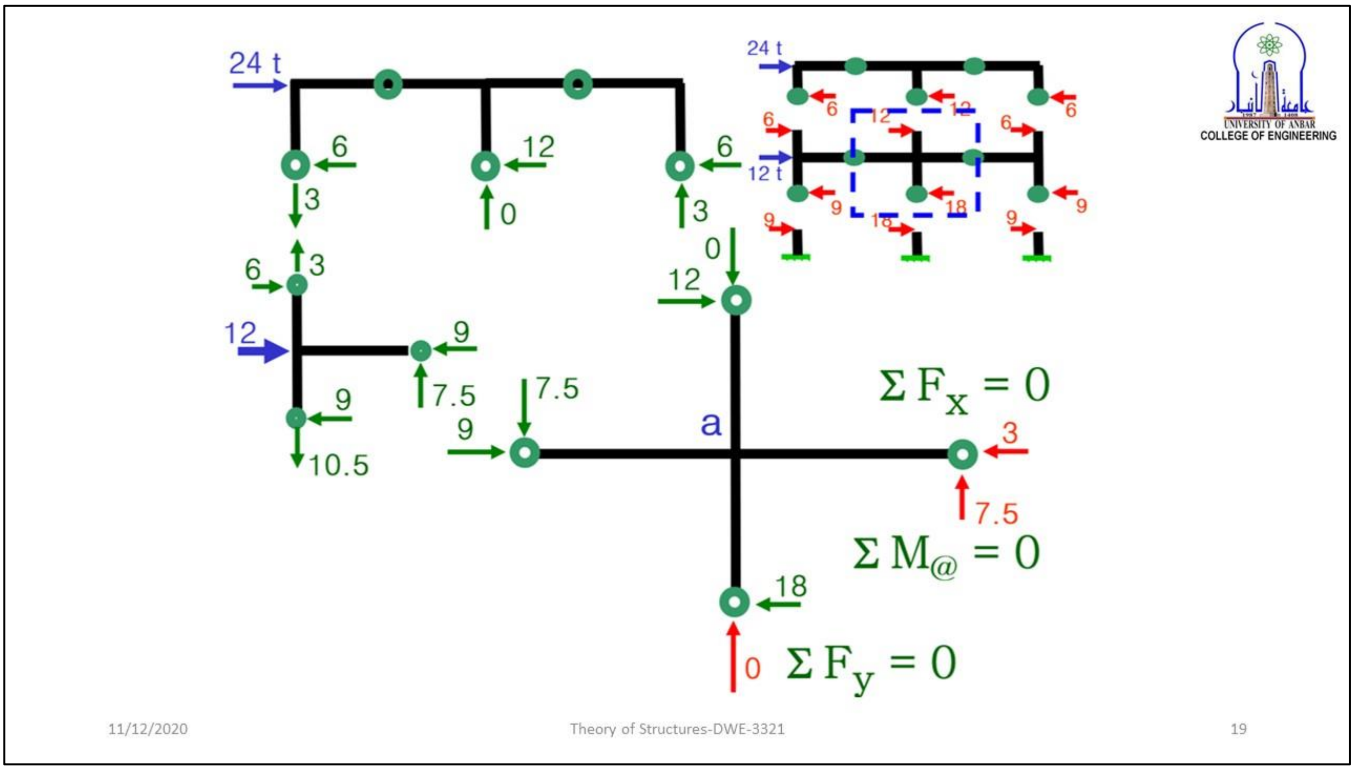
24 t
6
12
6
3
0
3
6
3
12
9
7.5
7.5
9
9
10.5
0
12
a
3
7.5
18
0
$\Sigma F_x = 0$
$\Sigma M_@ = 0$
$\Sigma F_y = 0$
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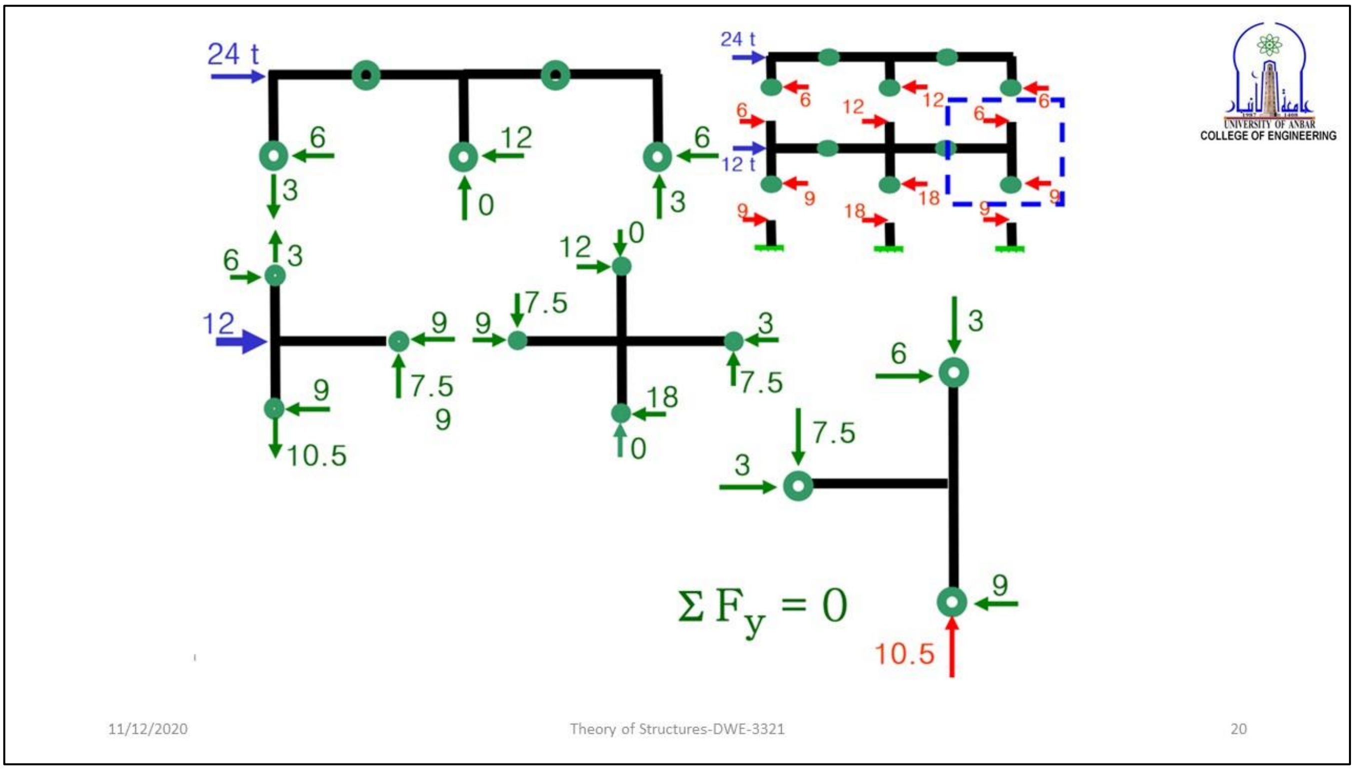
24 t
6
12
6
3
0
3
6
3
12
9
7.5
9
9
10.5
12
0
9
7.5
3
7.5
18
0
3
6
7.5
3
9
10.5
$\Sigma F_y = 0$
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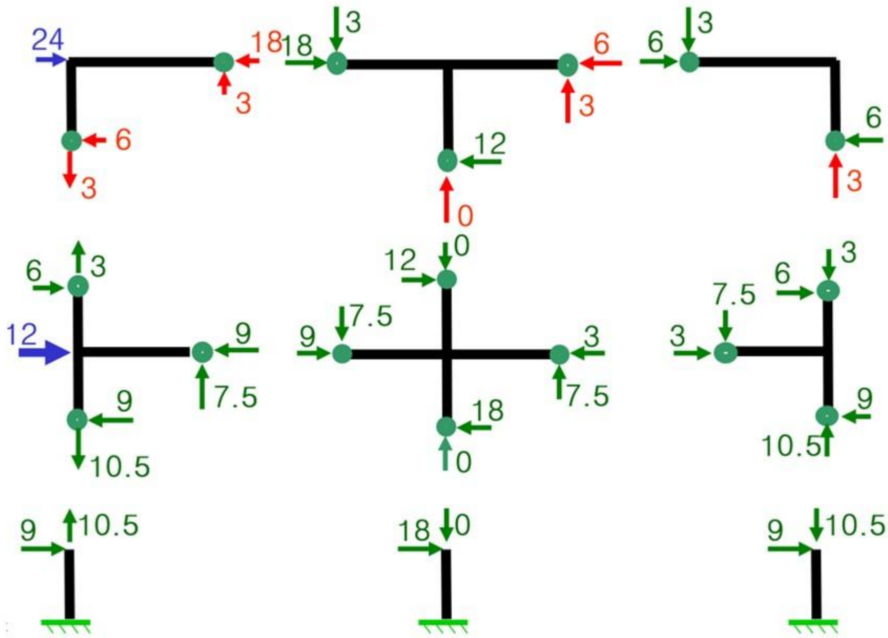

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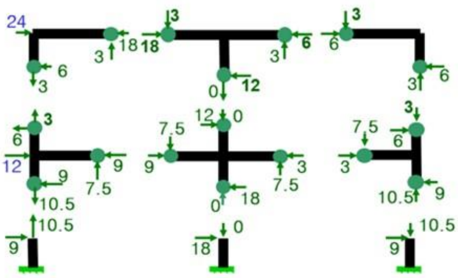

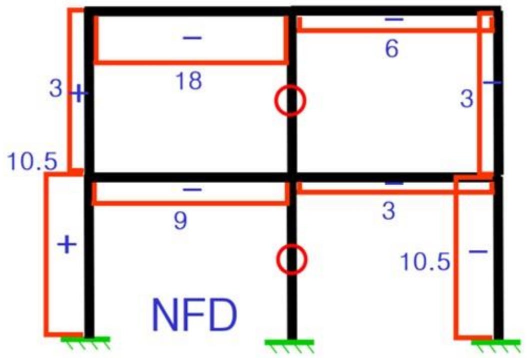

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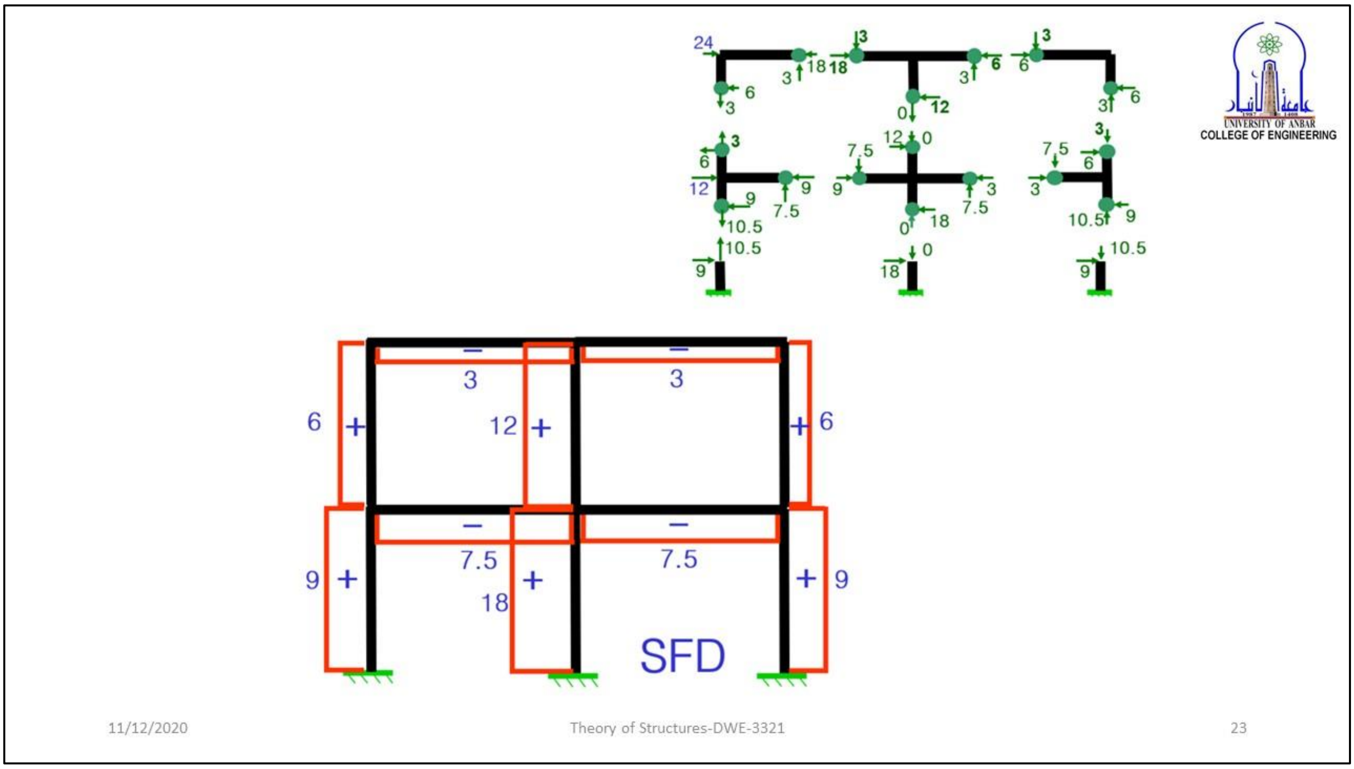
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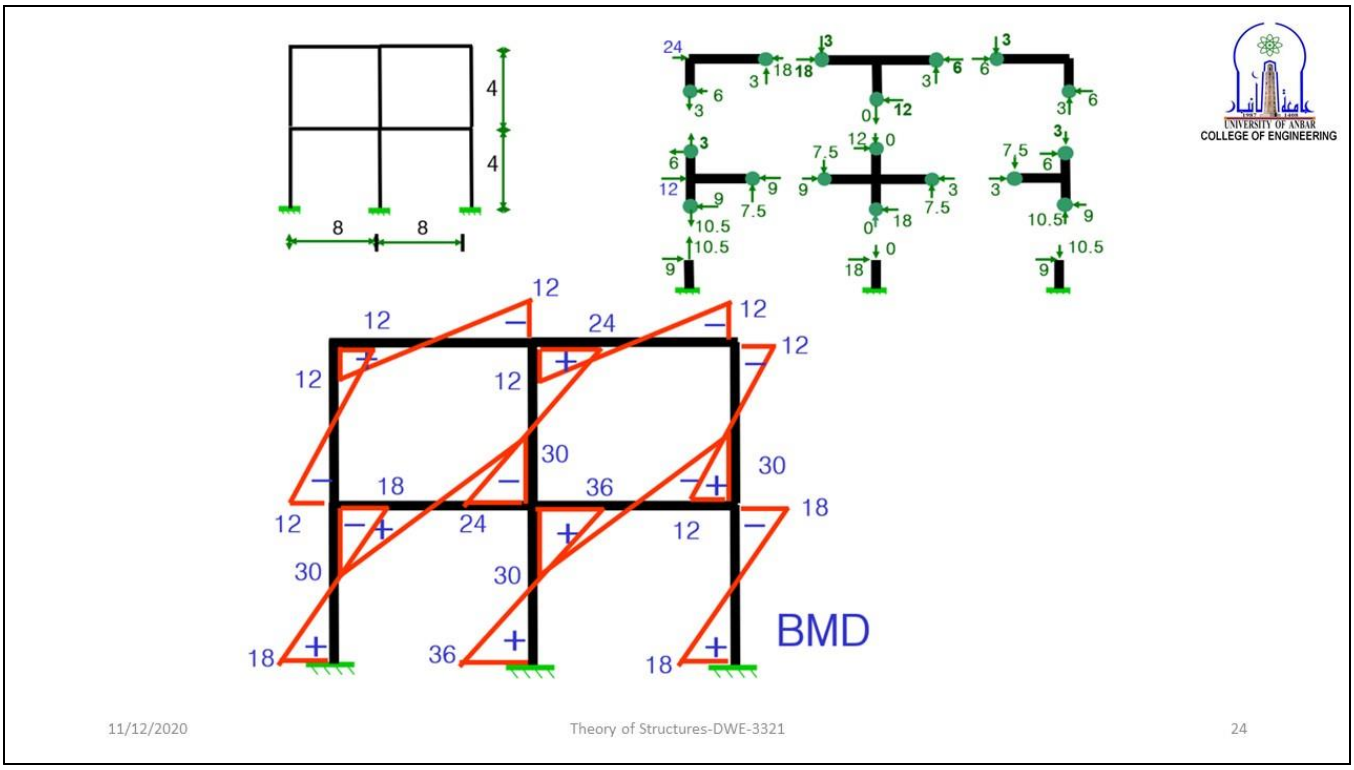
23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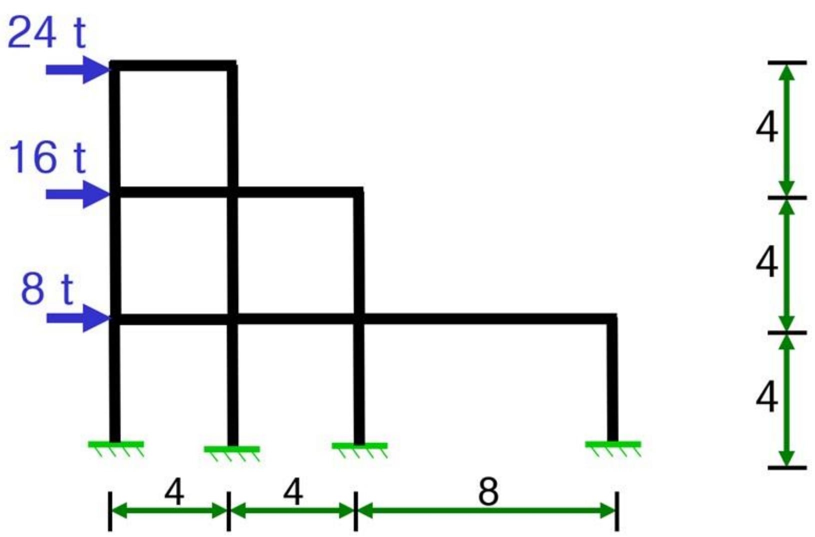
Example-2:



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Solve it yourself based on what you have learned in the lecture!

It is the same question for the tutorial session

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Unit-5

Influence Lines

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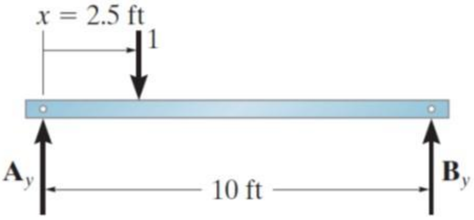
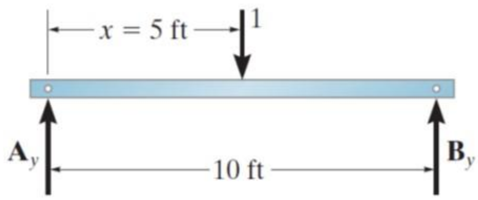
Example:



$\circlearrowleft + \Sigma M_B = 0; -A_y\,(10) + 1\,(7.5) = 0$

$A_y = 0.75$

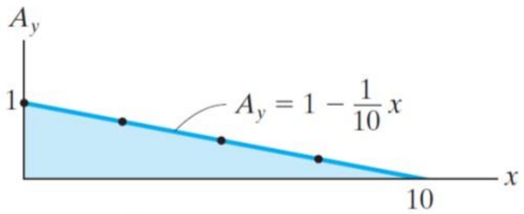


$\circlearrowleft + \Sigma M_B = 0; -A_y\,(10) + 1\,(5) = 0$

$A_y = 0.5$

x	A_y
0	1
2.5	0.75
5	0.5
7.5	0.25
10	0

(d)



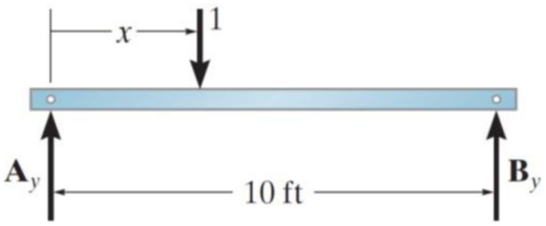
influence line for A_y

(e)

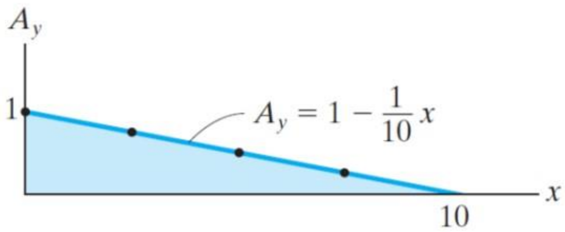
Influence line equation:

$\circlearrowleft + \Sigma M_B = 0; \quad -A_y(10) + (10 - x)(1) = 0$

$A_y = 1 - \frac{1}{10}x$



(f)



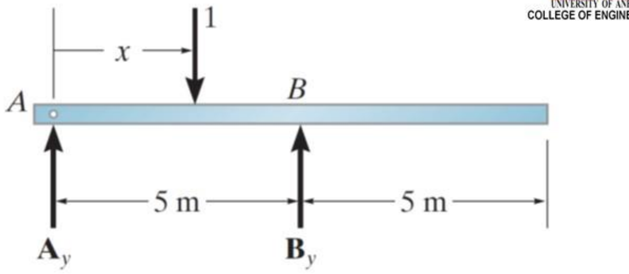
influence line for A_y

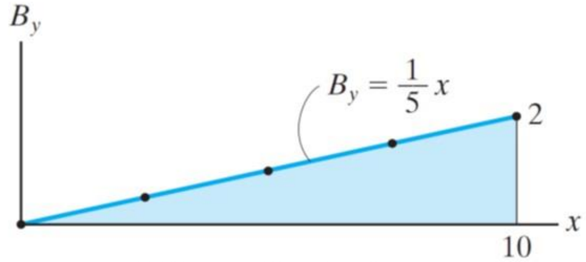
(e)

Example: Construct the influence line for the vertical reaction at B of the beam in the figure.

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(5) - 1(x) = 0$$

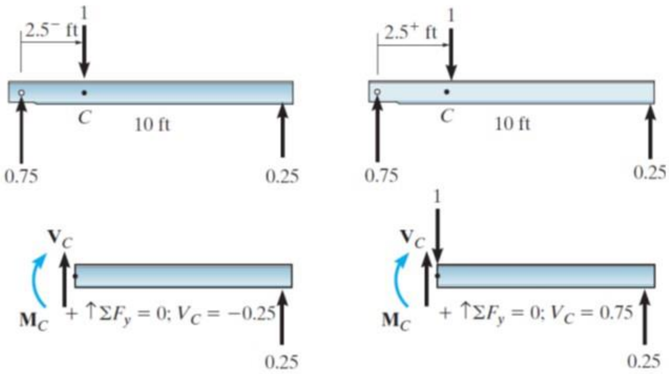
$$B_y = \tfrac{1}{5}x$$

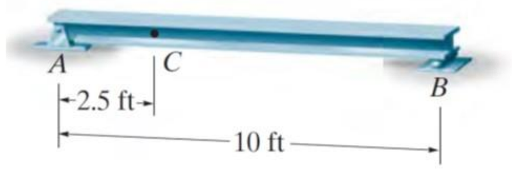




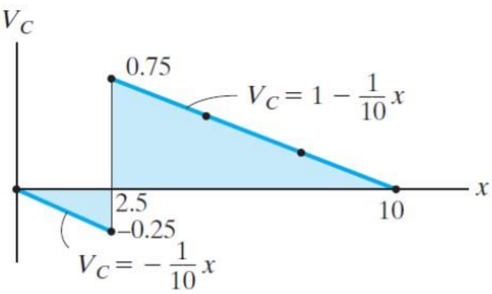
influence line for B_y

Example: Construct the influence line for the shear at point C of the beam in the figure.

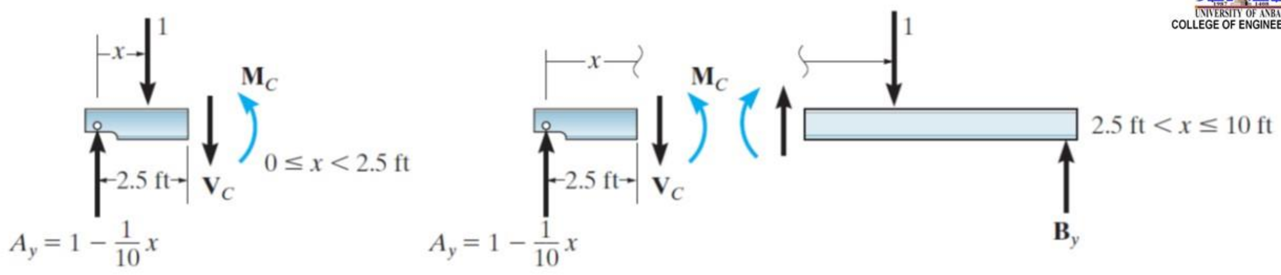


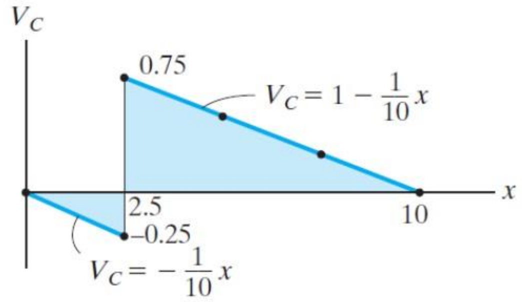


x	V_C
0	0
2.5^-	−0.25
2.5^+	0.75
5	0.5
7.5	0.25
10	0



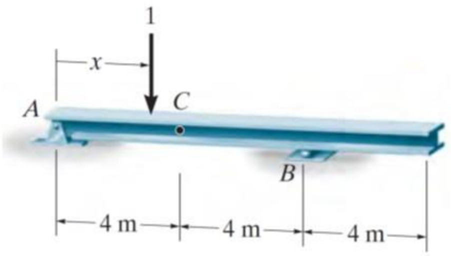
Influence line equation:

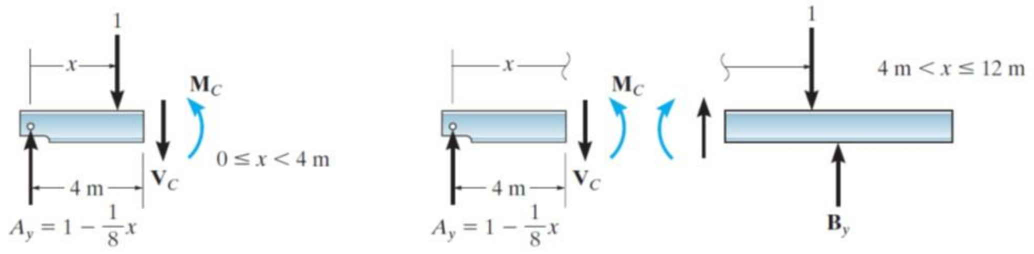




Example: Construct the influence line for the shear at point C of the beam in the figure.

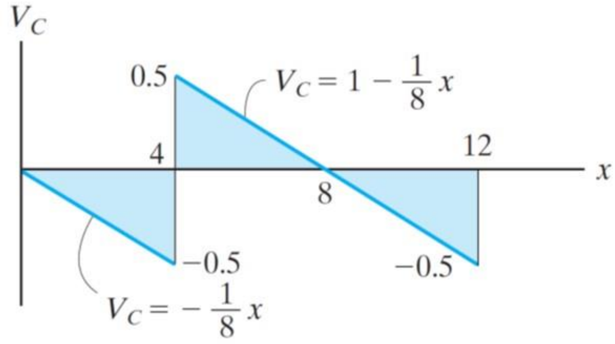
x	V_C
0	0
4^-	−0.5
4^+	0.5
8	0
12	−0.5



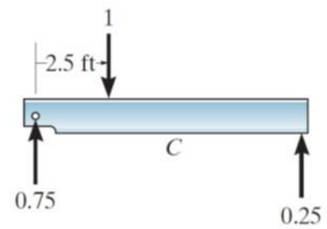


$$V_C = -\tfrac{1}{8}x \qquad 0 \le x < 4\ \text{m}$$

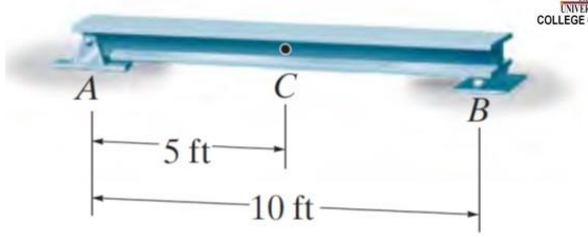
$$V_C = 1 - \tfrac{1}{8}x \qquad 4\ \text{m} < x \le 12\ \text{m}$$

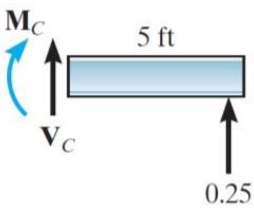


Example: Construct the influence line for the moment at point C of the beam in the figure.

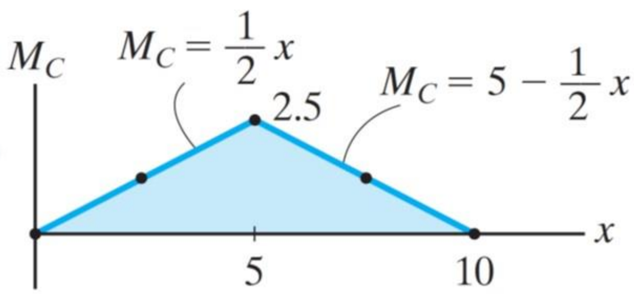


x	M_C
0	0
2.5	1.25
5	2.5
7.5	1.25
10	0





$$\circlearrowleft + \Sigma M_C = 0;\ -M_C + 0.25\,(5) = 0$$
$$M_C = 1.25$$



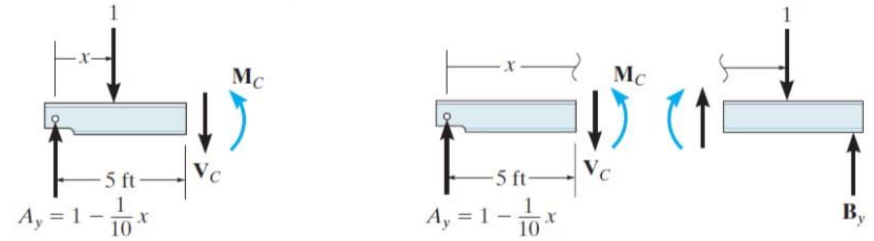
Influence line equation:

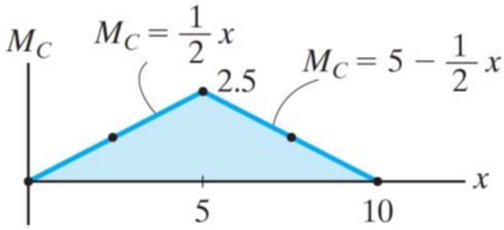
$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 1(5 - x) - \left(1 - \frac{1}{10}x\right)5 = 0$

$M_C = \frac{1}{2}x \qquad 0 \le x < 5\text{ ft}$

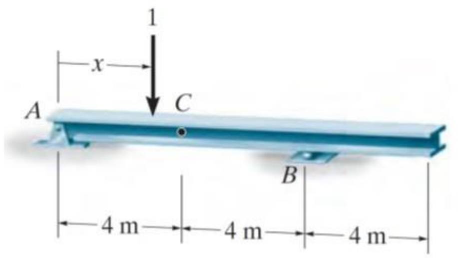
$\circlearrowleft + \Sigma M_C = 0; \quad M_C - \left(1 - \frac{1}{10}x\right)5 = 0$

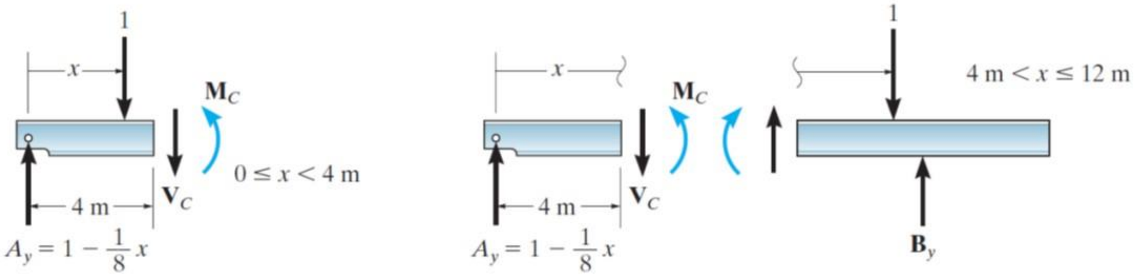
$M_C = 5 - \frac{1}{2}x \qquad 5\text{ ft} < x \le 10\text{ ft}$





Example: Construct the influence line for the moment at point C of the beam in the figure.



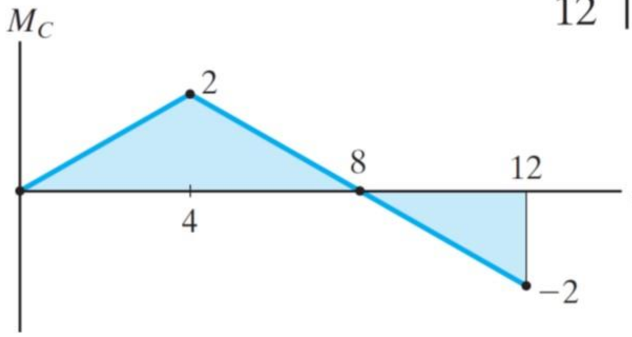


Influence line equation:

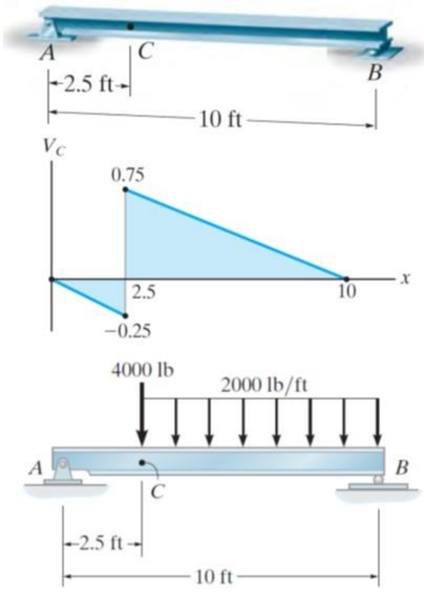
$$M_C = \tfrac{1}{2}x \qquad 0 \le x < 4\text{ m}$$

$$M_C = 4 - \tfrac{1}{2}x \qquad 4\text{ m} < x \le 12\text{ m}$$

x	M_C
0	0
4	2
8	0
12	−2



Example: Determine the maximum positive shear that can be developed at point **C** in the beam shown in the figure due to a concentrated moving load of **4000 lb** and a uniform moving load of **2000 lb/ft**.



Concentrated force:

$$V_C = 0.75(4000\text{ lb}) = 3000\text{ lb}$$

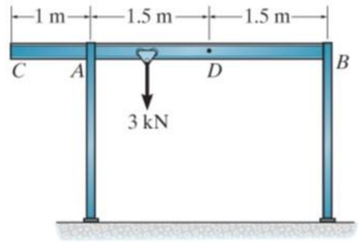
Uniform load:

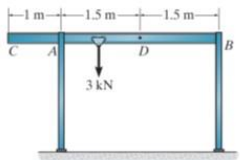
$$V_C = \left[\tfrac{1}{2}(10\text{ ft} - 2.5\text{ ft})(0.75)\right]2000\text{ lb/ft} = 5625\text{ lb}$$

Total maximum load:

$$(V_C)_{\max} = 3000\text{ lb} + 5625\text{ lb} = 8625\text{ lb}$$

Example: The frame structure shown in the figure is used to support a hoist for transferring loads for storage at points underneath it. It is anticipated that the load on the dolly is **3 kN** and the beam CB has a mass of **24 kg/m**. Assume **A** is a pin and **B** is a roller. Determine the maximum vertical support reactions at **A** and **B** and the maximum moment in the beam at **D**.

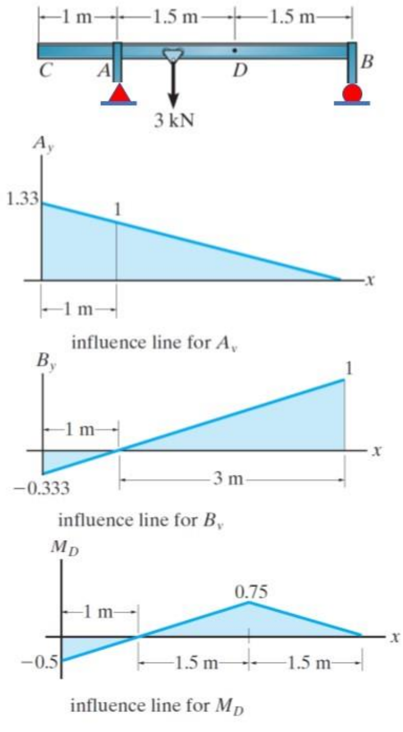




$$(A_y)_{max} = 3000(1.33) + 24(9.81)\left[\tfrac{1}{2}(4)(1.33)\right]$$
$$= 4.63 \text{ kN}$$

$$(B_y)_{max} = 3000(1) + 24(9.81)\left[\tfrac{1}{2}(3)(1)\right] + 24(9.81)\left[\tfrac{1}{2}(1)(-0.333)\right]$$
$$= 3.31 \text{ kN}$$ *Ans.*

$$(M_D)_{max} = 3000(0.75) + 24(9.81)\left[\tfrac{1}{2}(1)(-0.5)\right] + 24(9.81)\left[\tfrac{1}{2}(3)(0.75)\right]$$
$$= 2.46 \text{ kN} \cdot \text{m}$$ *Ans.*



Qualitative Influence Lines

- The Muller-Breslau principle states:

  The ***influence line*** for a function (reaction, shear, moment) is to the same scale as the deflected shape of the beam when the beam is acted on by the function.

  To draw the deflected shape properly, the ability of the beam to resist the applied function must be removed.

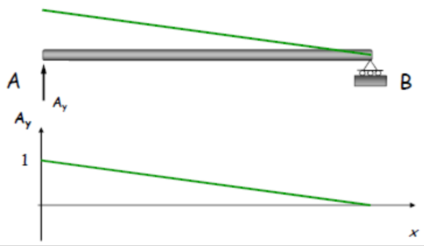
Qualitative Influence Lines

- For example, consider the following simply supported beam.

- Let's try to find the shape of the influence line for the vertical reaction at A.

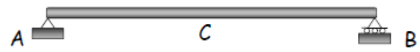
Qualitative Influence Lines

- Remove the ability to resist movement in the vertical direction at A by using a guided roller



Qualitative Influence Lines

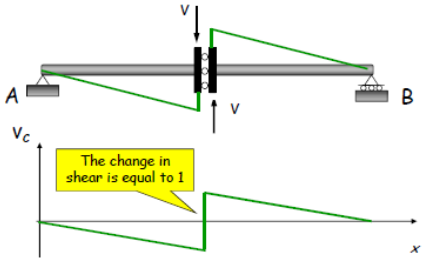
- Consider the following simply supported beam.



- Let's try to find the shape of the influence line for the shear at the mid-point (point C).

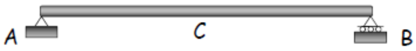
Qualitative Influence Lines

- Remove the ability to resist shear at point C



Qualitative Influence Lines

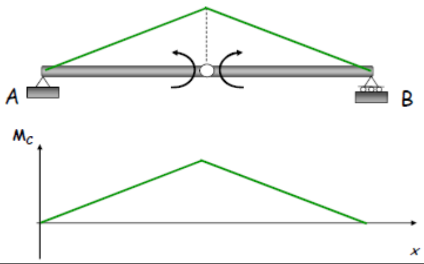
- Consider the following simply supported beam.



- Let's try to find the shape of the influence line for the moment at the mid-point (point C).

Qualitative Influence Lines

- Remove the ability to resist moment at C by using a hinge

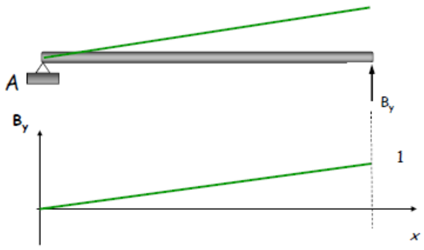


Qualitative Influence Lines

- Sketch the shape of the influence line for the reaction at point B

Qualitative Influence Lines

- Sketch the shape of the influence line for the reaction at point B

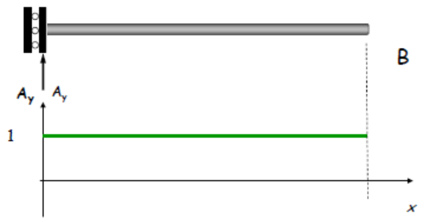


Qualitative Influence Lines

- Sketch the shape of the influence line for the reaction at point A

Qualitative Influence Lines

- Sketch the shape of the influence line for the reaction at point *A*

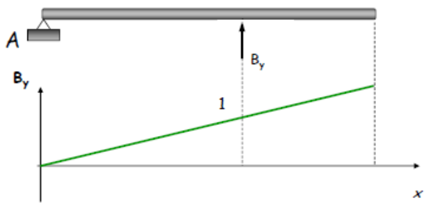


Qualitative Influence Lines

- Sketch the shape of the influence line for the reaction at point B

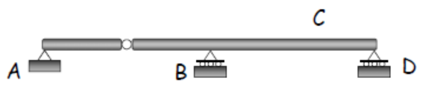
Qualitative Influence Lines

- Sketch the shape of the influence line for the reaction at point B



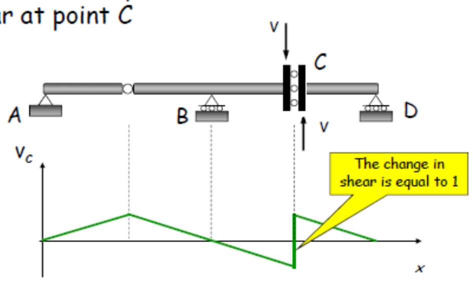
Qualitative Influence Lines

- Sketch the shape of the influence line for the shear at point *C*



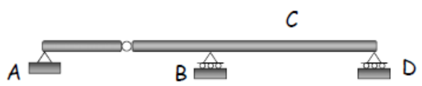
Qualitative Influence Lines

- Sketch the shape of the influence line for the shear at point *C*



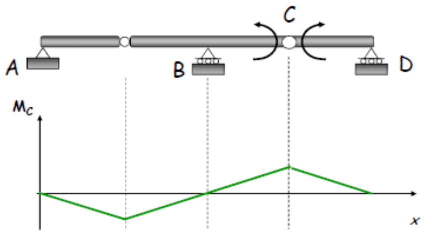
Qualitative Influence Lines

- Sketch the shape of the influence line for the moment at point *C*



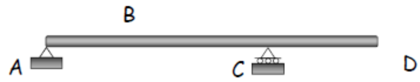
Qualitative Influence Lines

- Sketch the shape of the influence line for the moment at point C



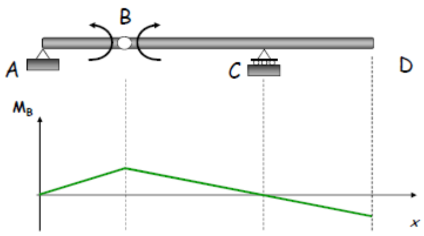
Qualitative Influence Lines

- Sketch the shape of the influence line for the moment at point B



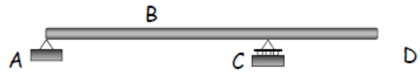
Qualitative Influence Lines

- Sketch the shape of the influence line for the moment at point B



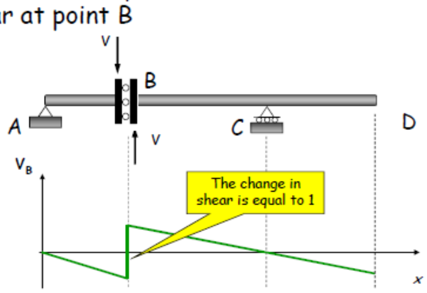
Qualitative Influence Lines

- Sketch the shape of the influence line for the shear at point B



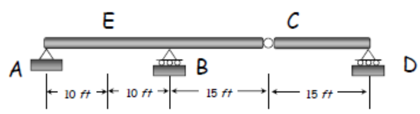
Qualitative Influence Lines

- Sketch the shape of the influence line for the shear at point B



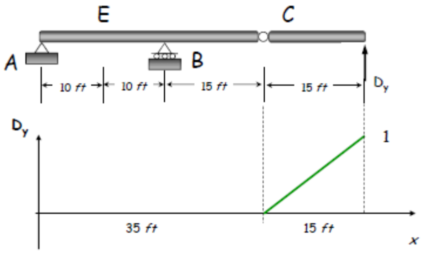
Qualitative Influence Lines

- Draw the influence lines for the vertical reaction at D and the shear at E.



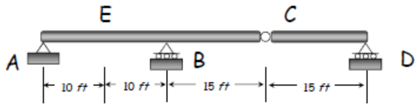
Qualitative Influence Lines

- Draw the influence lines for the vertical reaction at D and the shear at E.



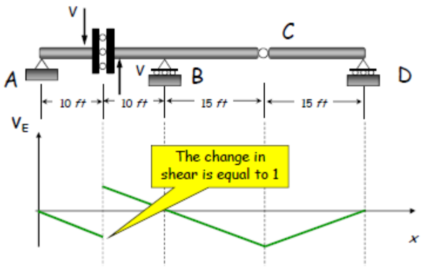
Qualitative Influence Lines

- Draw the influence lines for the vertical reaction at D and the shear at E.



Qualitative Influence Lines

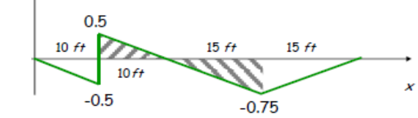
- Draw the influence lines for the vertical reaction at D and the shear at E.



Qualitative Influence Lines

- Draw the influence lines for the vertical reaction at D and the shear at E.
- The change in shear at point E is equal to 1
- The influence lines can be determined by similar triangles the values of

$$\frac{0.5}{10ft} = \frac{x}{15ft} \Rightarrow x = \frac{0.5(15ft)}{10ft} = 0.75$$



Influence Lines for Floor Girders

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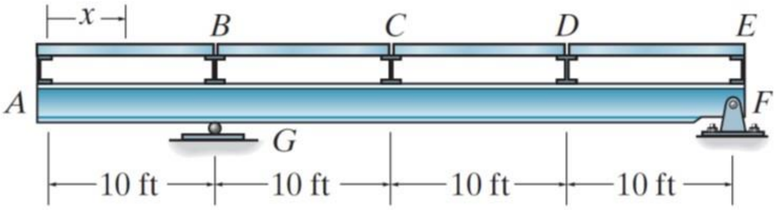
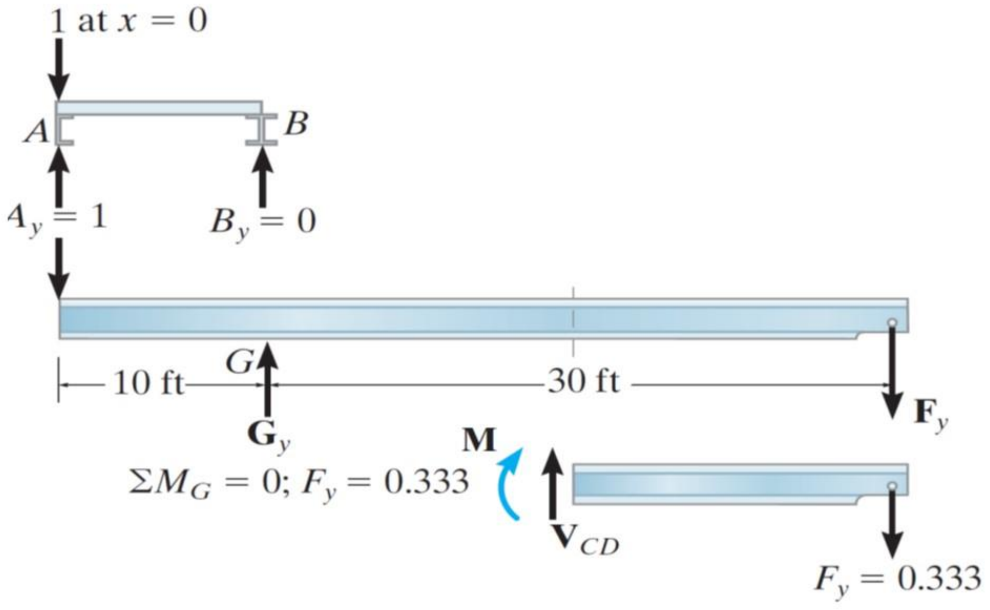
Example: Draw the influence line for the shear in panel ***CD*** of the floor girder in the figure.

x	V_{CD}
0	0.333
10	0
20	−0.333
30	0.333
40	0





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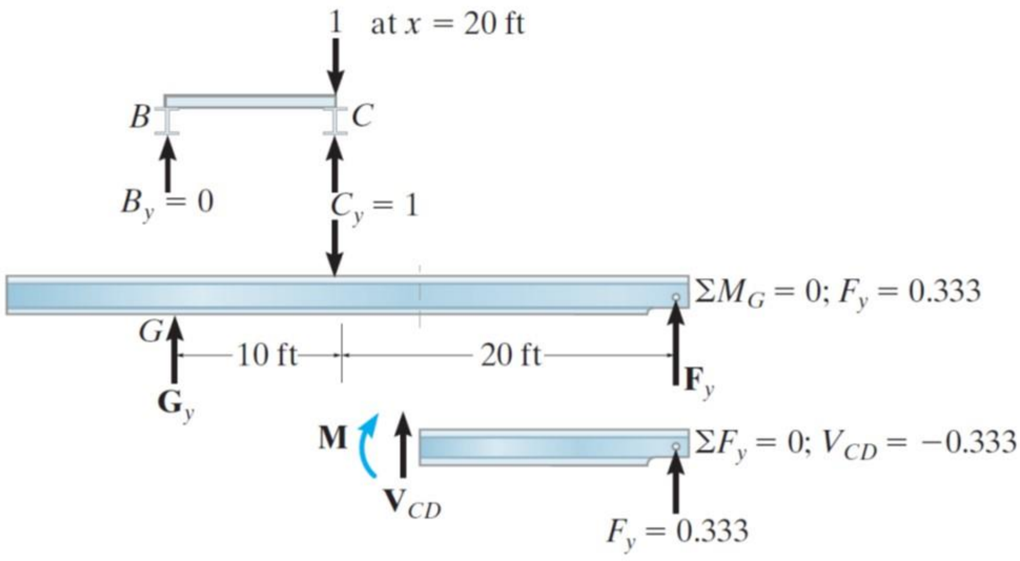

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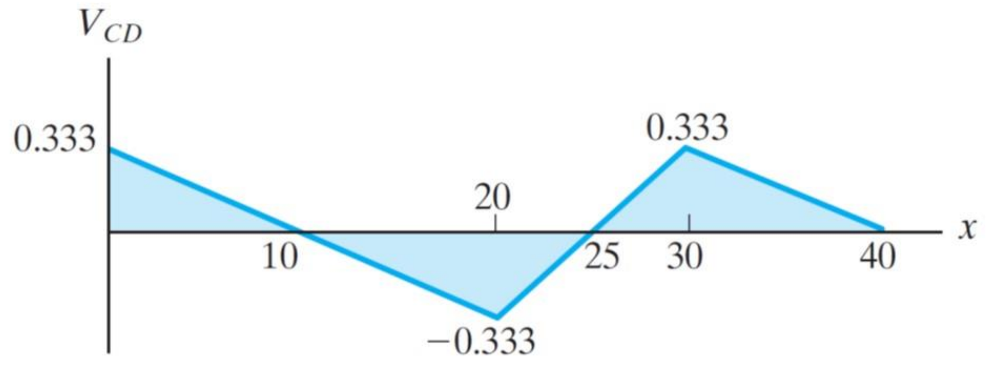

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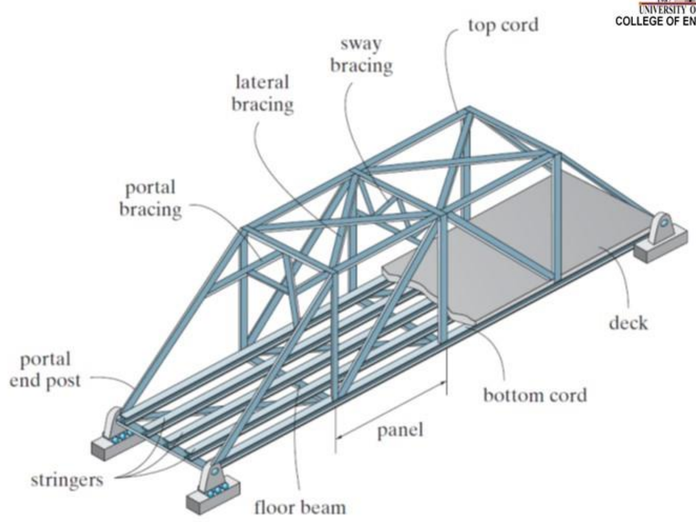
Influence Lines for Trusses

11/13/2020



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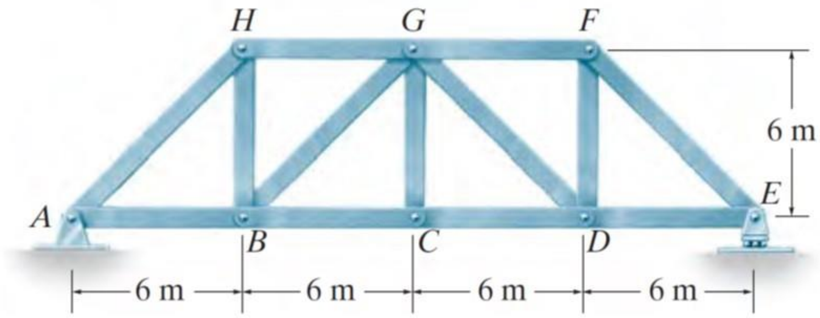
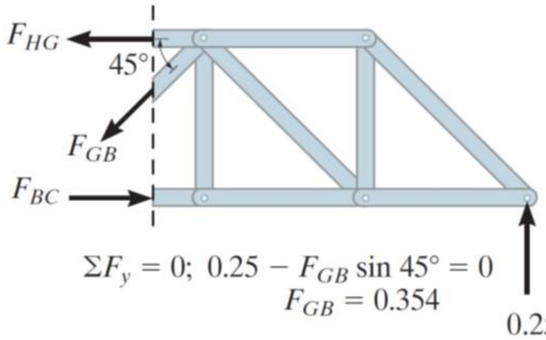
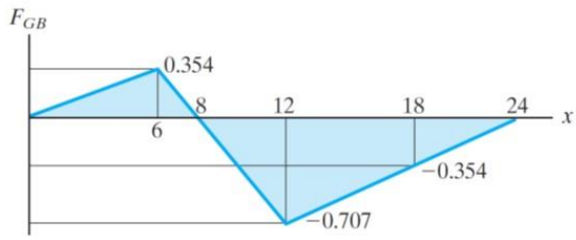
Example: Draw the influence line for the force in members **GB** and **CG** of the bridge truss shown in the figure.





$$\Sigma F_y = 0;\ 0.25 - F_{GB} \sin 45° = 0$$
$$F_{GB} = 0.354$$

x	F_{GB}
0	0
6	0.354
12	−0.707
18	−0.354
24	0



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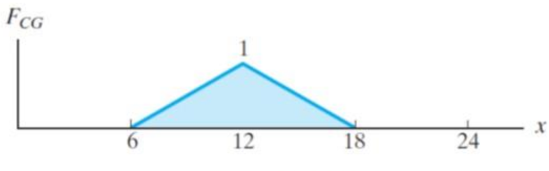
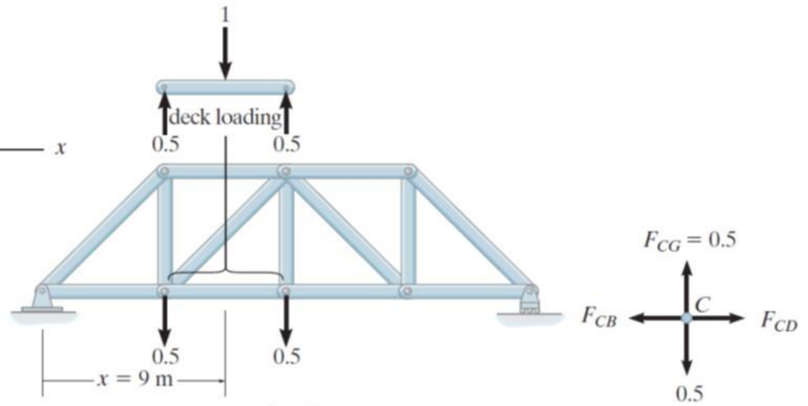


influence line for F_{CG}
(d)



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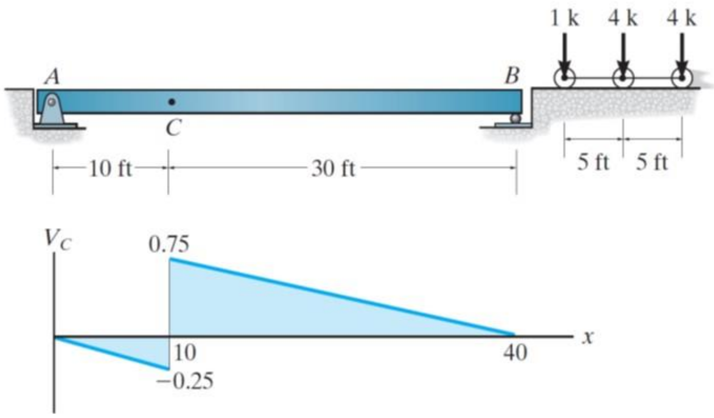
Maximum Influence at Point Due to Series of Concentrated Loads



Case 1: $(V_C)_1 = 1(0.75) + 4(0.625) + 4(0.5) = 5.25\text{ k}$

Case 2: $(V_C)_2 = 1(-0.125) + 4(0.75) + 4(0.625) = \boxed{5.375\text{ k}}$

Case 3: $(V_C)_3 = 1(0) + 4(-0.125) + 4(0.75) = 2.5\text{ k}$

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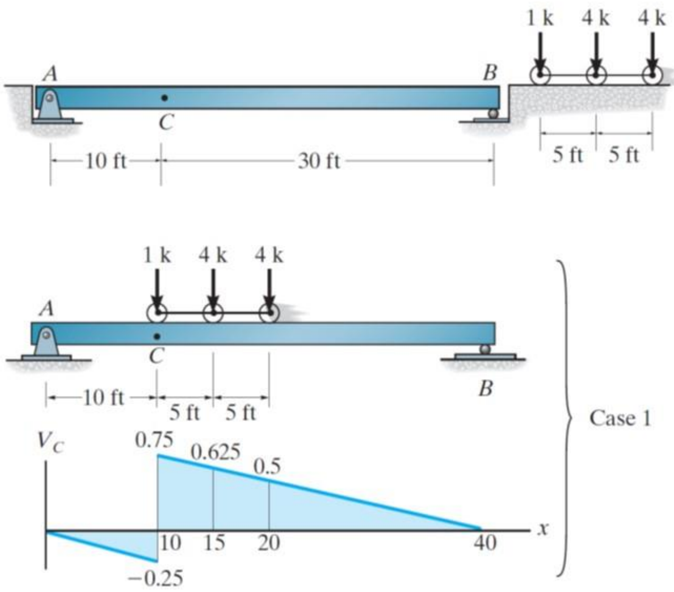

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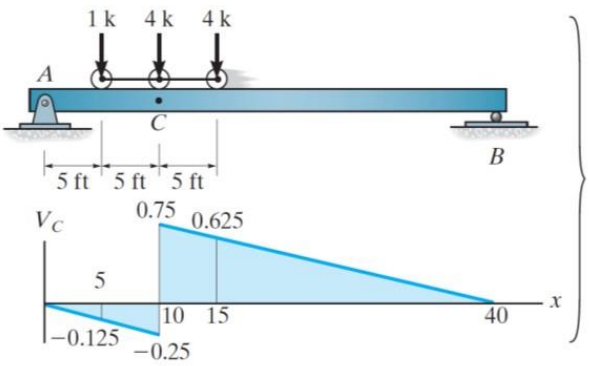
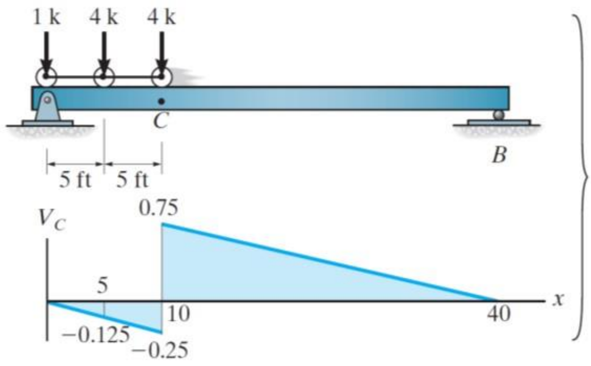


Case 2



Case 3

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When many concentrated loads act on the span, the trial-and-error computations used above can be tedious. Instead, the critical position of the loads can be determined in a more direct manner by finding the change in shear, which occurs when the loads are moved from Case 1 to Case 2, then from Case 2 to Case 3, and so on. As long as each computed is positive, the new position will yield a larger shear in the beam at C than the previous position. Each movement is investigated until a negative change in shear is computed.

$\Delta V = Ps(x_2 - x_1)$ Sloping Line	$\Delta V = P(y_2 - y_1)$ Jump

$$\Delta V_{1-2} = 1(-1) + [1 + 4 + 4](0.025)(5) = +0.125 \text{ k}$$

$$\Delta V_{2-3} = 4(-1) + (1 + 4 + 4)(0.025)(5) = -2.875 \text{ k}$$

Since ΔV2-3 is negative, Case 2 is the position of the critical loading, as determined previously.

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Moment :

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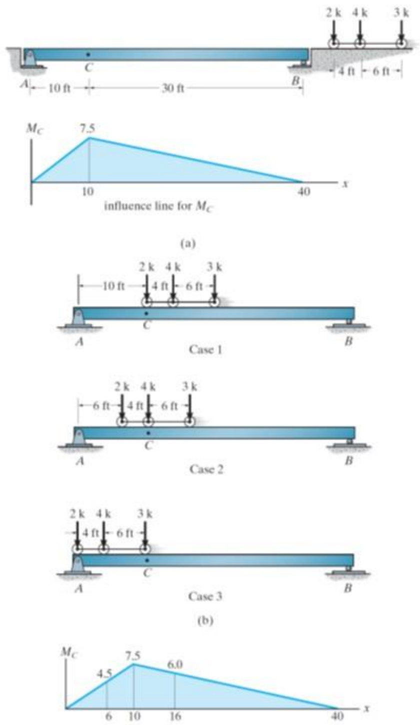
$$\Delta M = Ps(x_2 - x_1)$$
Sloping Line

$$\Delta M_{1-2} = -2\left(\frac{7.5}{10}\right)(4) + (4+3)\left(\frac{7.5}{40-10}\right)(4) = 1.0\ \text{k}\cdot\text{ft}$$

$$\Delta M_{2-3} = -(2+4)\left(\frac{7.5}{10}\right)(6) + 3\left(\frac{7.5}{40-10}\right)(6) = -22.5\ \text{k}\cdot\text{ft}$$

$$(M_C)_{\max} = 2(4.5) + 4(7.5) + 3(6.0) = 57.0\ \text{k}\cdot\text{ft}$$



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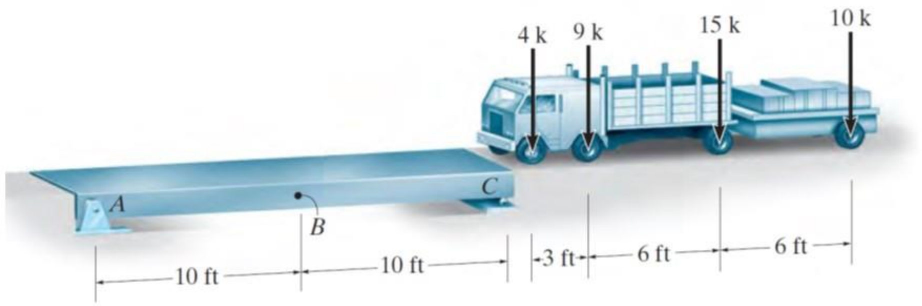
Example: Determine the maximum positive shear created at point **B** in the beam shown in figure due to the wheel loads of the moving truck.



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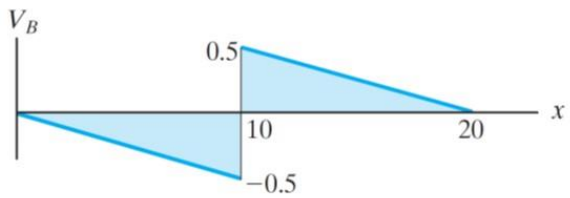
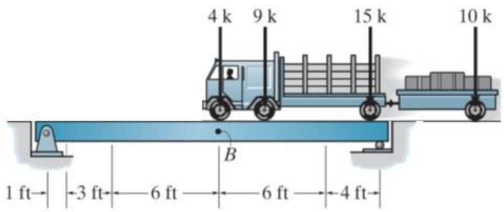
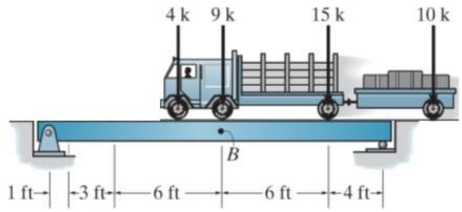
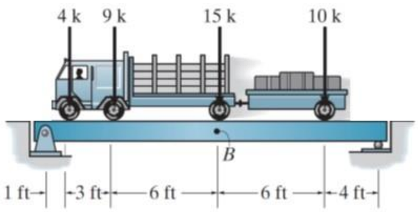




$$\Delta V_B = 4(-1) + (4 + 9 + 15)\left(\frac{0.5}{10}\right)3 = +0.2\ \text{k}$$



$$\Delta V_B = 9(-1) + (4 + 9 + 15)\left(\frac{0.5}{10}\right)(6) + 10\left(\frac{0.5}{10}\right)(4) = +1.4\ \text{k}$$



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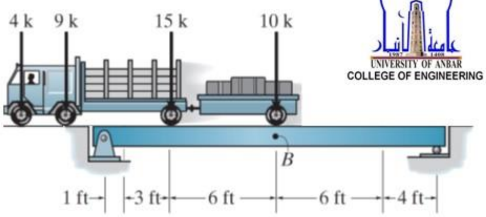
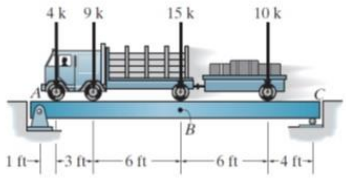
$$\Delta V_B = 15(-1) + 4\left(\frac{0.5}{10}\right)(1) + 9\left(\frac{0.5}{10}\right)(4) + (15+10)\left(\frac{0.5}{10}\right)(6)$$
$$= -5.5 \text{ k}$$

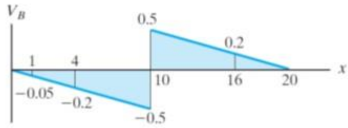


Since ΔVB is negative, Previous case is the position of the critical loading

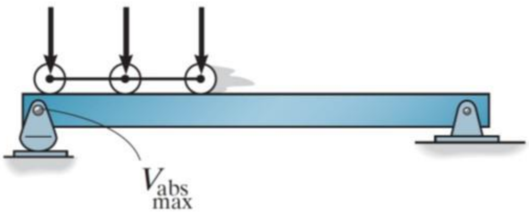
$$(V_B)_{max} = 4(-0.05) + 9(-0.2) + 15(0.5) + 10(0.2)$$
$$= 7.5 \text{ k} \qquad \textit{Ans.}$$

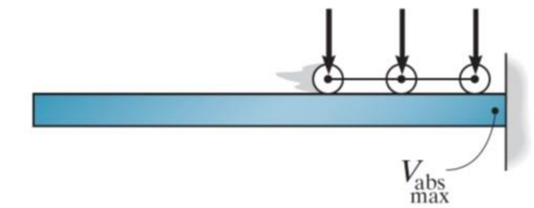


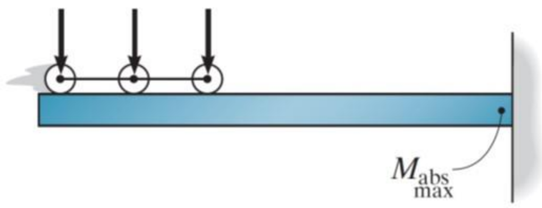
In practice one also has to consider motion of the truck from left to right and then choose the maximum value between these two situations.

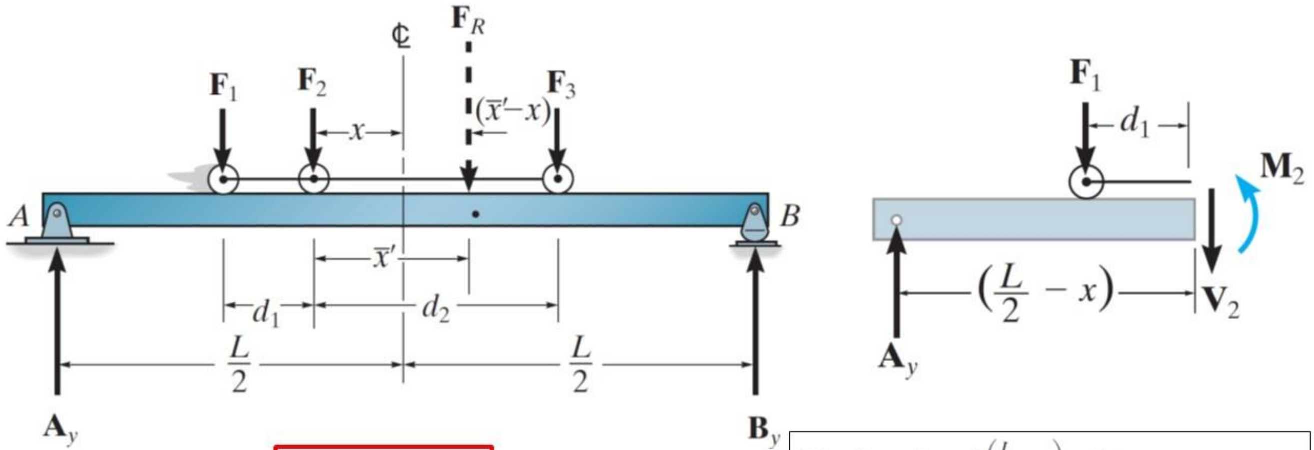


Absolute Maximum Shear and Moment









$$\boxed{x = \frac{\overline{x}'}{2}}$$

$\Sigma M = 0;\qquad M_2 = A_y\left(\frac{L}{2} - x\right) - F_1 d_1$

$$= \frac{1}{L}(F_R)\left[\frac{L}{2} - (\overline{x}' - x)\right]\left(\frac{L}{2} - x\right) - F_1 d_1$$

$$= \frac{F_R L}{4} - \frac{F_R \overline{x}'}{2} - \frac{F_R x^2}{L} + \frac{F_R x \overline{x}'}{L} - F_1 d_1$$

For maximum M_2 we require

$$\frac{dM_2}{dx} = \frac{-2F_R x}{L} + \frac{F_R \overline{x}'}{L} = 0$$

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Example : Determine the absolute maximum moment in the simply supported bridge deck shown in the figure.

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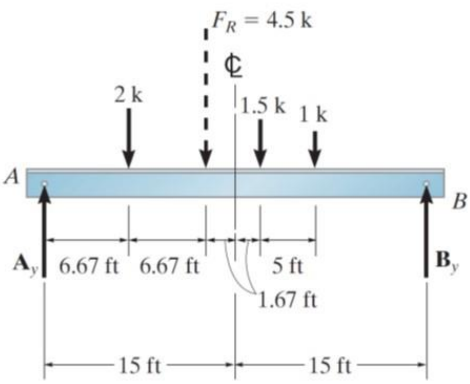

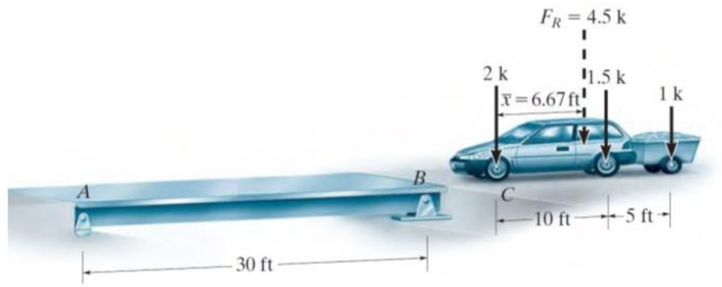

$$+\downarrow F_R = \Sigma F;\qquad F_R = 2 + 1.5 + 1 = 4.5\text{ k}$$

$$\curvearrowleft + M_{R_C} = \Sigma M_C;\qquad 4.5\overline{x} = 1.5(10) + 1(15)$$

$$\overline{x} = 6.67\text{ ft}$$

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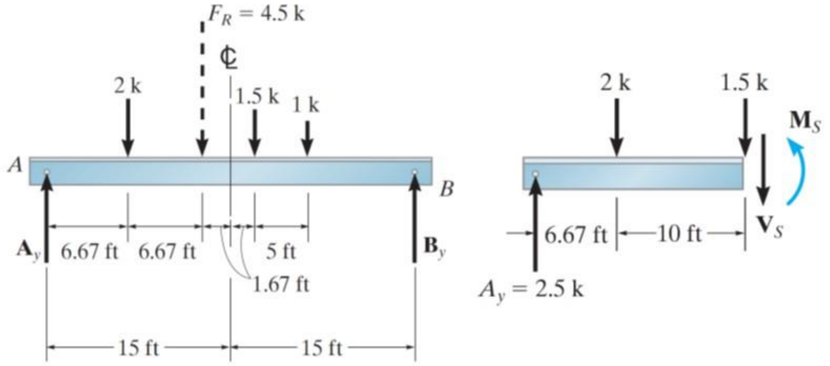
Case-1 :



$$\circlearrowright + \Sigma M_B = 0; \qquad -A_y(30) + 4.5(16.67) = 0 \qquad A_y = 2.50 \text{ k}$$

$$\circlearrowright + \Sigma M_S = 0; \qquad -2.50(16.67) + 2(10) + M_S = 0$$

$$M_S = 21.7 \text{ k} \cdot \text{ft}$$

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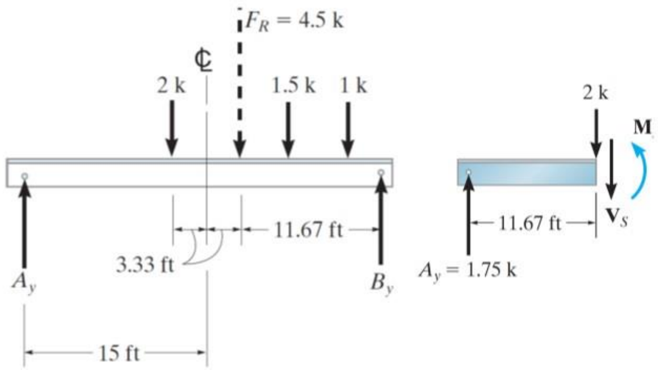
Case-2 :

$$M_S = 20.4 \text{ k} \cdot \text{ft}$$



By Comparison the maximum moment is :

$$M_S = 21.7 \text{ k} \cdot \text{ft}$$

Which occurs under the 1.5 k load when positioned as in the case-1

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Unit-6

Deflection of Statically Determinate Structures

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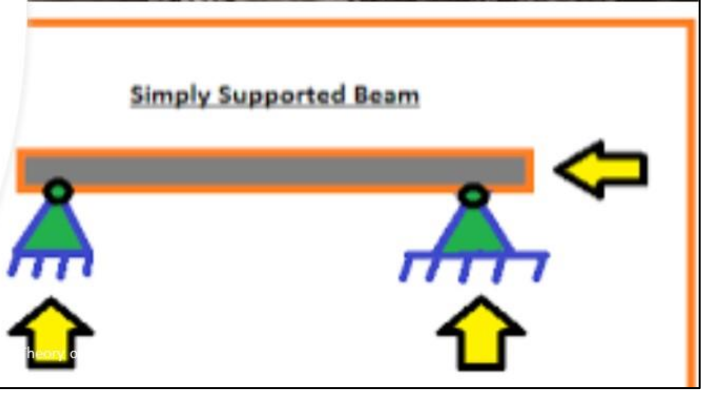
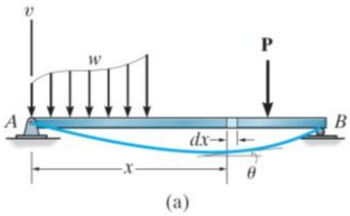
Determinate OR Indeterminate



Elastic Beam Theory :

$$\boxed{\frac{d^2v}{dx^2} = \frac{M}{EI}}$$



1- Double integration method. X

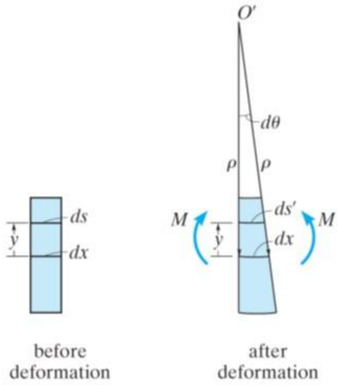
2- Moment-area theorem. X

3- Conjugate-beam method. X

4- Energy methods:

- Method of virtual work. √

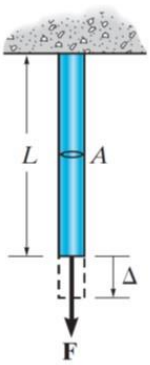
- Castigliano theorem. √



External Work and Strain Energy

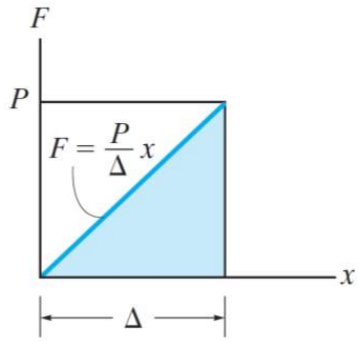
$$\boxed{U_e = U_i}$$

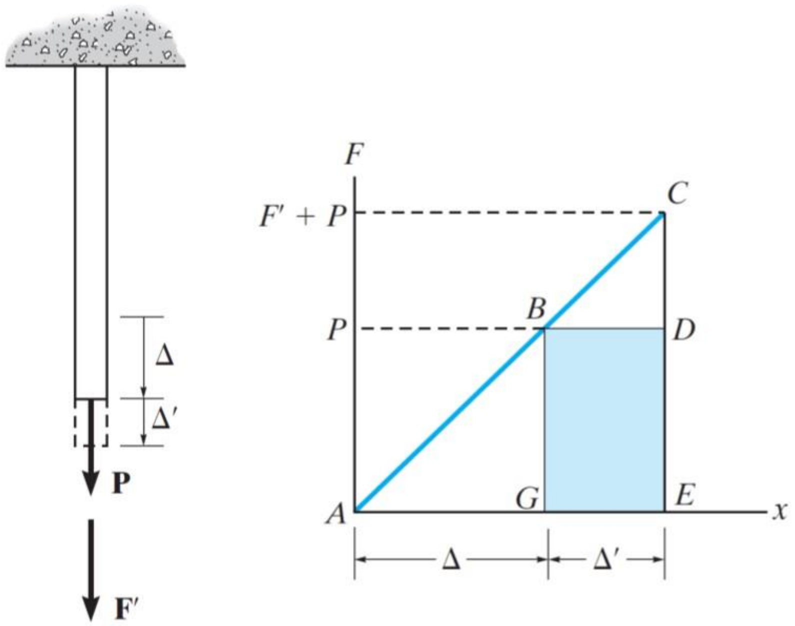
External Work – Force :



$$U_e = \int_0^x F\,dx$$

$$U_e = \tfrac{1}{2}P\Delta$$

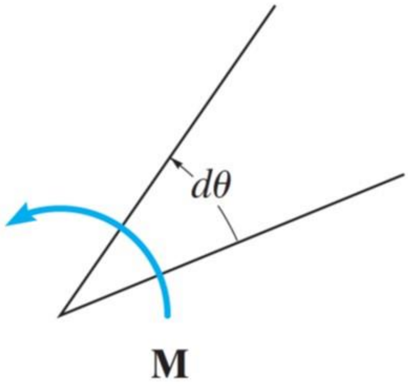




$$U_e' = P\Delta'$$

External Work – Moment :

$$U_e = \int_0^{\theta} M\, d\theta$$



$$U_e = \frac{1}{2} M\theta$$

$$U_e' = M\theta'$$

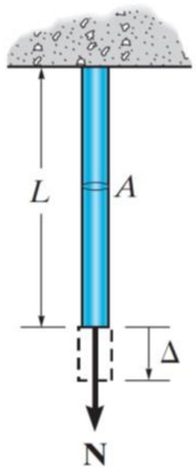
Strain Energy – Axial Force :

$$\Delta = \frac{NL}{AE}$$

$$\boxed{U_e = U_i}$$

$$U_e = \frac{1}{2}P\Delta$$

$$U_i = \frac{N^2L}{2AE}$$

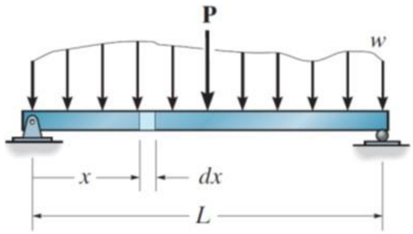


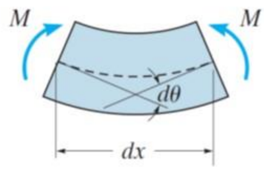
Strain Energy – Bending :

$$d\theta = (M/EI)\,dx$$

$$dU_i = \frac{M^2\,dx}{2EI}$$

$$U_i = \int_0^L \frac{M^2\,dx}{2EI}$$





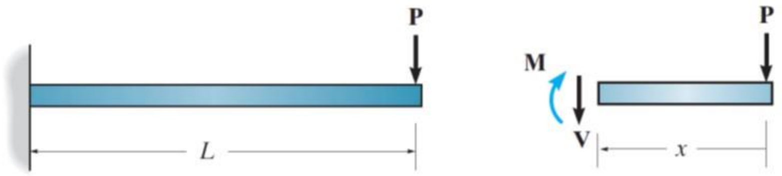
Principle of Work and Energy

$$U_i = \int_0^L \frac{M^2\,dx}{2EI} = \int_0^L \frac{(-Px)^2\,dx}{2EI} = \frac{1}{6}\,\frac{P^2L^3}{EI}$$

$$U_e = U_i$$

$$\frac{1}{2}P\Delta = \frac{1}{6}\frac{P^2L^3}{EI}$$

$$\Delta = \frac{PL^3}{3EI}$$



Principle of Virtual Work

$$\Sigma P\Delta = \Sigma u\delta$$

Work of External Loads = Work of Internal Loads

$$1 \cdot \Delta = \Sigma u \cdot dL$$

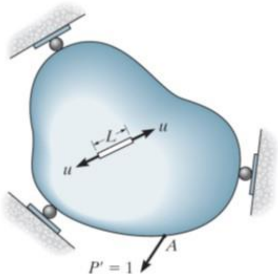
virtual loadings

real displacements

$$1 \cdot \theta = \Sigma u_\theta \cdot dL$$

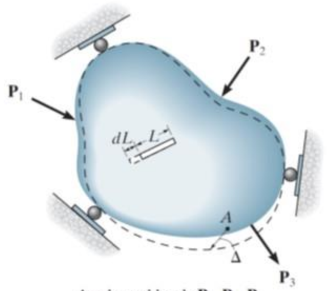
virtual loadings

real displacements



Apply virtual load $P' = 1$

(a)



Apply real loads $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$

(b)

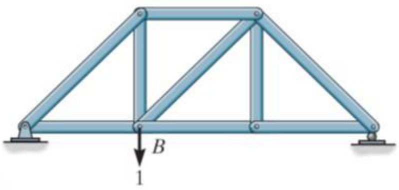
Method of Virtual Work

1- Trusses

A- External Loading: $$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

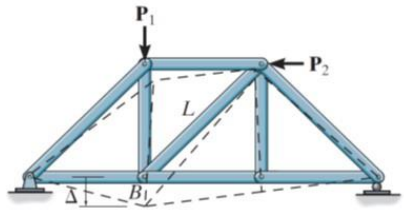
B- Temperature Effect: $$1 \cdot \Delta = \Sigma n\alpha\, \Delta T\, L$$

C- Fabrication Error: $$1 \cdot \Delta = \Sigma n\, \Delta L$$



Apply virtual unit load to B

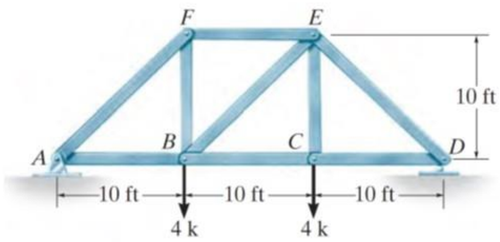
(a)

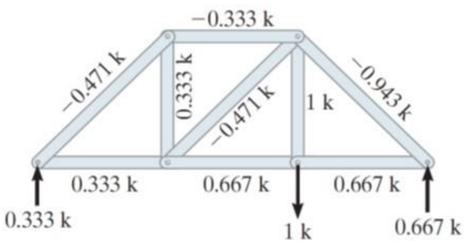


Apply real loads $\mathbf{P}_1$, $\mathbf{P}_2$

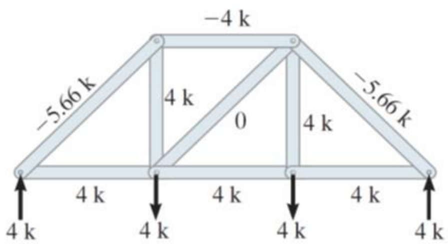
(b)

Example: Determine the vertical displacement of joint **C** of the steel truss shown in the figure. The cross-sectional area of each member is **A** = 0.5 in^2 and **E** = 29x10^3 ksi.





virtual forces **n**



real forces **N**

Member	n (k)	N (k)	L (ft)	nNL ($k^2 \cdot ft$)
AB	0.333	4	10	13.33
BC	0.667	4	10	26.67
CD	0.667	4	10	26.67
DE	−0.943	−5.66	14.14	75.42
FE	−0.333	−4	10	13.33
EB	−0.471	0	14.14	0
BF	0.333	4	10	13.33
AF	−0.471	−5.66	14.14	37.71
CE	1	4	10	40
				$\Sigma 246.47$

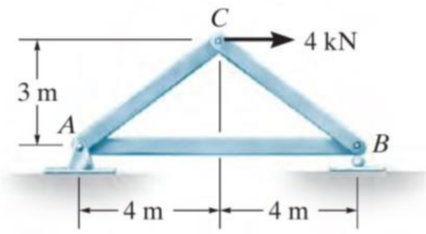
$$1\text{ k}\cdot\Delta_{C_v} = \sum\frac{nNL}{AE} = \frac{246.47\text{ k}^2\cdot\text{ft}}{AE} \qquad 1\text{ k}\cdot\Delta_{C_v} = \frac{(246.47\text{ k}^2\cdot\text{ft})(12\text{ in./ft})}{(0.5\text{ in}^2)(29(10^3)\text{ k/in}^2)}$$

$$\Delta_{C_v} = 0.204\text{ in.}$$

Example: The cross-sectional area of each member of the truss shown in the figure is **A** = 400 mm² and **E** = 200 GPa.

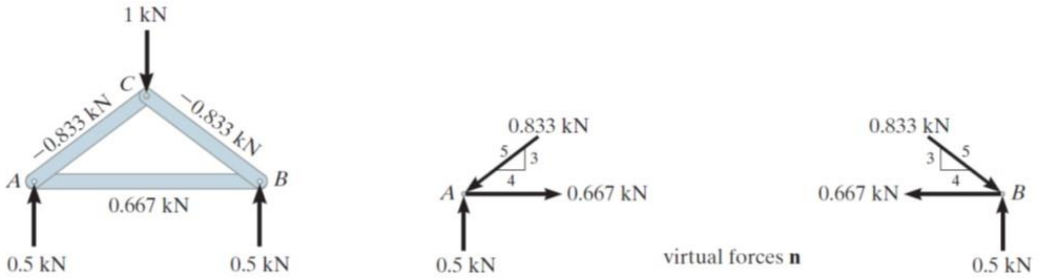
(a) Determine the vertical displacement of joint C if a 4-kN force is applied to the truss at C.

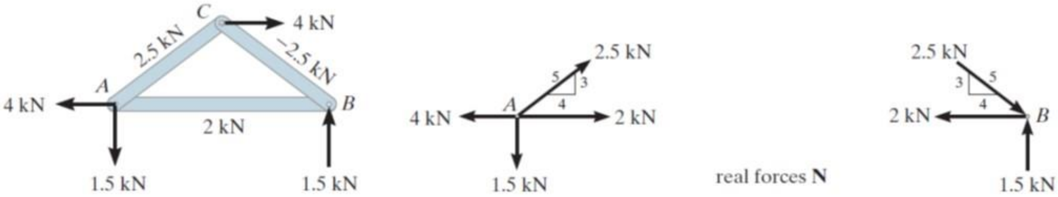
(b) If no loads act on the truss, what would be the vertical displacement of joint ***C*** if member *AB* were 5 mm too short?



Solution:

Part-A:





Member	n (kN)	N (kN)	L (m)	$n\,NL$ (kN$^2\cdot$m)
AB	0.667	2	8	10.67
AC	−0.833	2.5	5	−10.41
CB	−0.833	−2.5	5	10.41
				Σ10.67

$$1\text{ kN}\cdot\Delta_{C_v} = \sum\frac{nNL}{AE} = \frac{10.67\text{ kN}^2\cdot\text{m}}{AE}$$

$$1\text{ kN}\cdot\Delta_{C_v} = \frac{10.67\text{ kN}^2\cdot\text{m}}{400(10^{-6})\text{ m}^2(200(10^6)\text{ kN/m}^2)}$$

$$\Delta_{C_v} = 0.000133\text{ m} = 0.133\text{ mm}$$

Part-B:

$$1\cdot\Delta = \Sigma n\,\Delta L$$

$$1\text{ kN}\cdot\Delta_{C_v} = (0.667\text{ kN})(-0.005\text{ m})$$

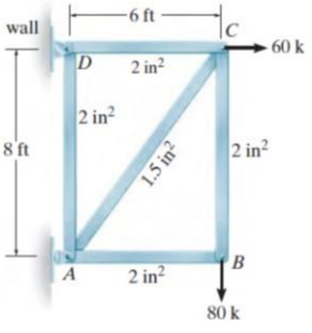
$$\Delta_{C_v} = -0.00333\text{ m} = -3.33\text{ mm}$$

Note:

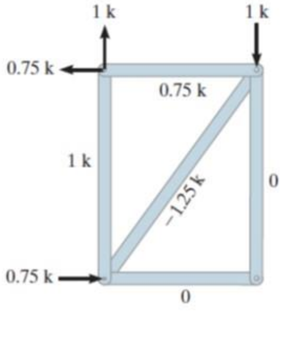
The negative sign indicates joint ***C*** is displaced *upward,* opposite to the 1-kN vertical load. Note that if the 4-kN load and fabrication error are both accounted for, the resultant displacement is then **Δcv** = 0.133 - 3.33 = -3.20 mm (upward).

Example: Determine the vertical displacement of joint **C** of the steel truss shown in the figure due to radiant heating from the wall, member **AD** is subjected to an increase in temperature of **ΔT** = +120° F. Take **α** = 0.6x10⁻⁵/°F and **E** = 29(10³) ksi. The cross-sectional area of each member is indicated in the figure.

Solution:

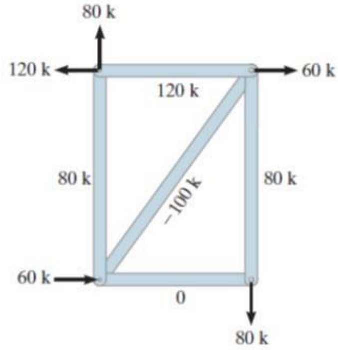


(a)



virtual forces **n**

(b)



real forces **N**

(c)

Solution:

$$1 \cdot \Delta_{C_v} = \sum \frac{nNL}{AE} + \Sigma n\alpha\, \Delta T\, L$$

$$= \frac{(0.75)(120)(6)(12)}{2[29(10^3)]} + \frac{(1)(80)(8)(12)}{2[29(10^3)]}$$

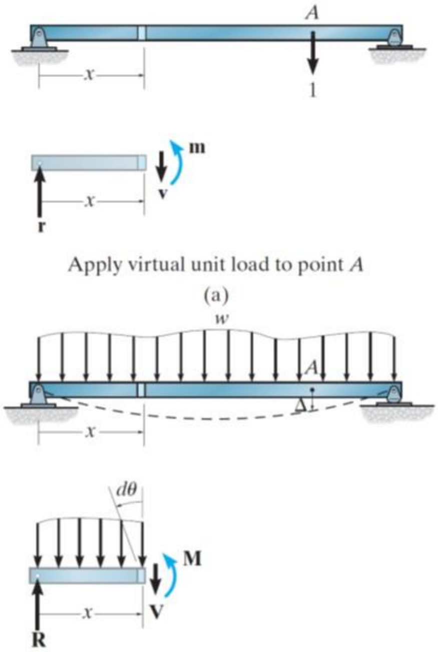
$$+ \frac{(-1.25)(-100)(10)(12)}{1.5[29(10^3)]} + (1)[0.6(10^{-5})](120)(8)(12)$$

$$\Delta_{C_v} = 0.658 \text{ in.}$$ *Ans.*

Method of Virtual Work

2- Beams and Frames:

$$\boxed{1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx}$$



Apply virtual unit load to point A

(a)

Apply real load w

(b)

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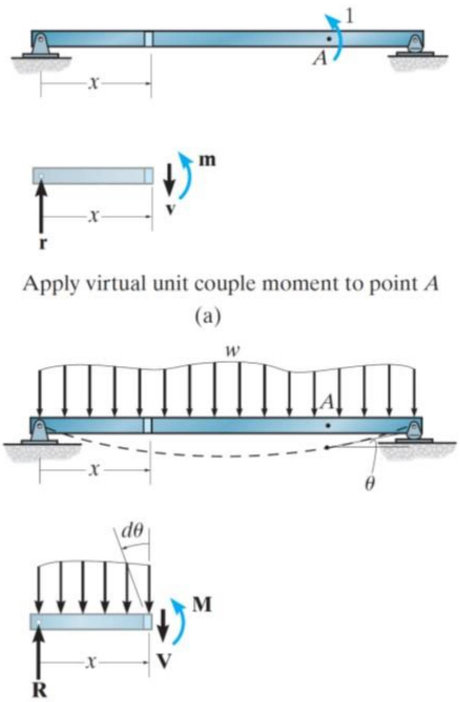
$$\boxed{1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx}$$



Apply virtual unit couple moment to point A

(a)

Apply real load w

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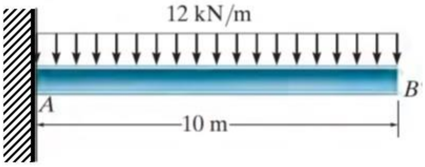
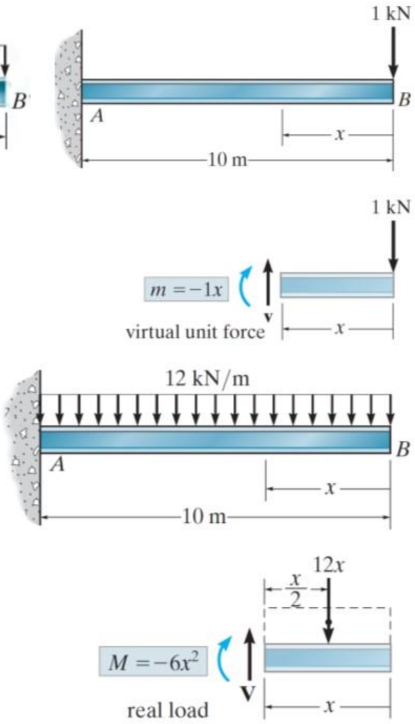
Example: Determine the displacement of point **B** of the steel beam shown in the figure. Take E = 200 GPa, I = 500x10⁶ mm⁴.



Solution:



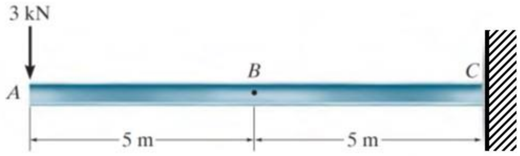
$$1\ \text{kN} \cdot \Delta_B = \int_0^L \frac{mM}{EI} dx = \int_0^{10} \frac{(-1x)(-6x^2)\,dx}{EI}$$

$$1\ \text{kN} \cdot \Delta_B = \frac{15(10^3)\ \text{kN}^2 \cdot \text{m}^3}{EI}$$

$$\Delta_B = \frac{15(10^3)\ \text{kN} \cdot \text{m}^3}{200(10^6)\ \text{kN/m}^2(500(10^6)\ \text{mm}^4)(10^{-12}\ \text{m}^4/\text{mm}^4)}$$

$$= 0.150\ \text{m} = 150\ \text{mm}$$

Example: Determine the slop **θ** at point **B** of the steel beam shown in the figure. Take E = 200 GPa, I = 600x10⁶ mm⁴.



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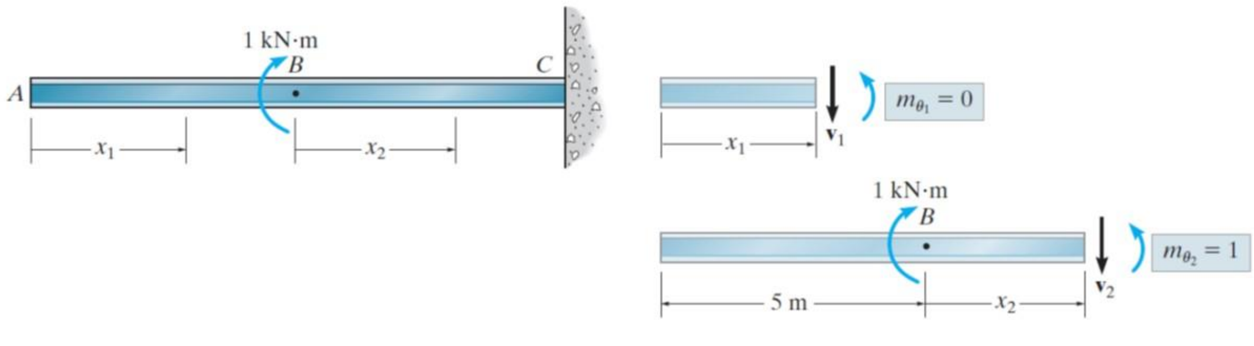
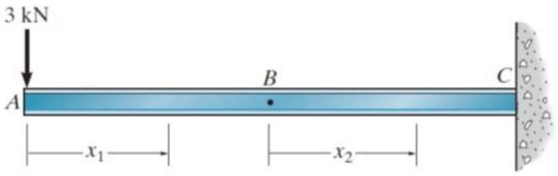
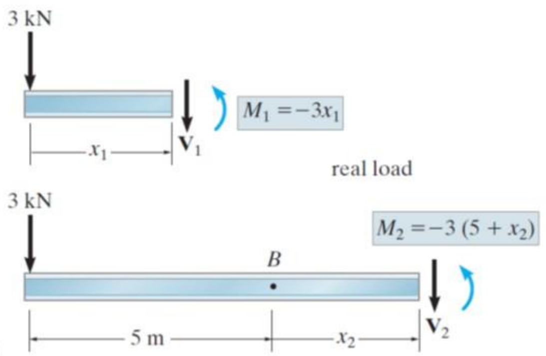
Solution:



virtual unit couple





$$1 \cdot \theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

$$= \int_0^5 \frac{(0)(-3x_1)\, dx_1}{EI} + \int_0^5 \frac{(1)[-3(5 + x_2)]\, dx_2}{EI}$$

$$\theta_B = \frac{-112.5 \text{ kN} \cdot \text{m}^2}{EI}$$

$$(1 \text{ kN} \cdot \text{m}) \cdot \theta_B = \frac{-112.5 \text{ kN}^2 \cdot \text{m}^3}{200(10^6) \text{ kN/m}^2[60(10^6) \text{ mm}^4](10^{-12} \text{ m}^4/\text{mm}^4)}$$

$$\theta_B = -0.00938 \text{ rad} \qquad \textit{Ans.}$$

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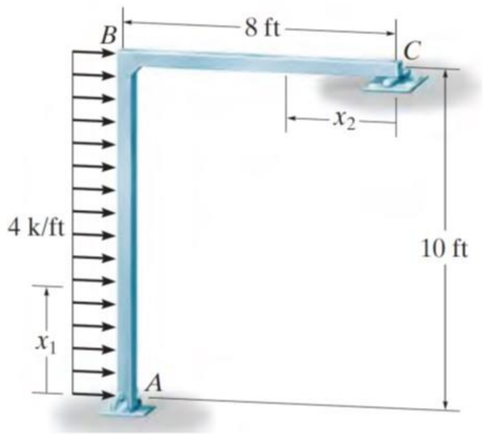
Example: Determine the horizontal displacement at point **C** of the frame shown in the figure. Take E = 29(10^3) ksi, I = 600 in^4 for both members.



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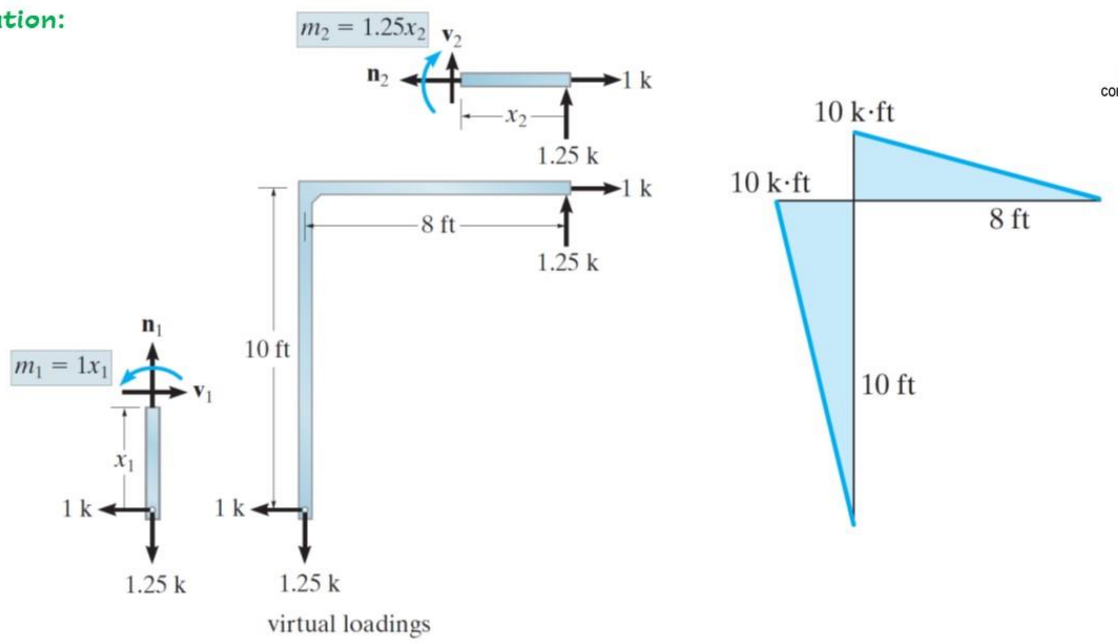
Solution:



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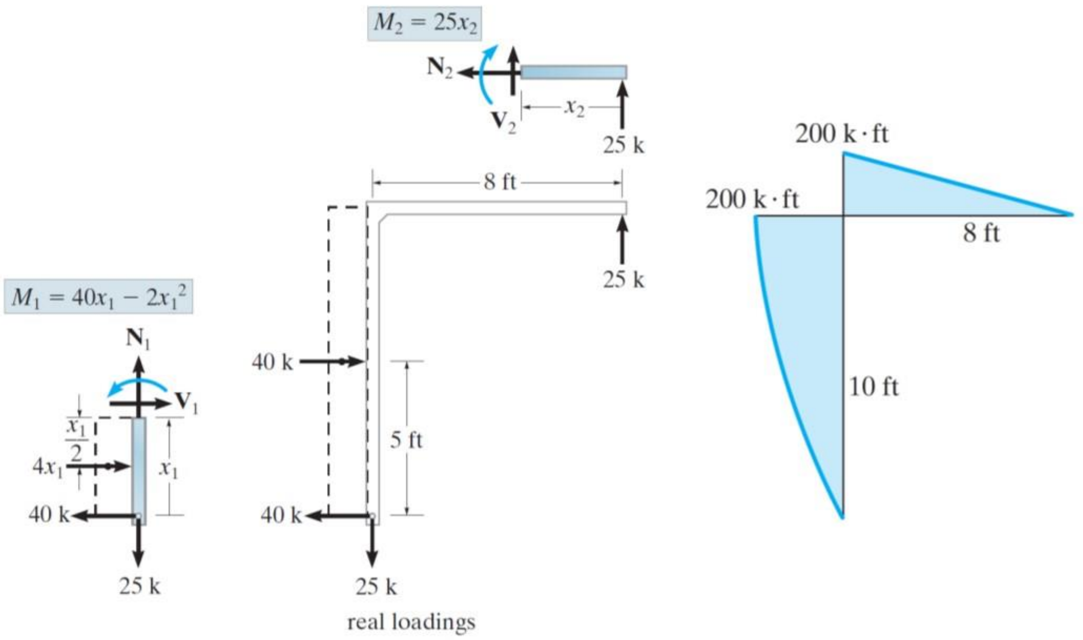

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$$1 \cdot \Delta_{C_h} = \int_0^L \frac{mM}{EI} dx = \int_0^{10} \frac{(1x_1)\left(40x_1 - 2x_1^2\right) dx_1}{EI} + \int_0^{8} \frac{(1.25x_2)(25x_2)\, dx_2}{EI}$$

$$\Delta_{C_h} = \frac{8333.3}{EI} + \frac{5333.3}{EI} = \frac{13\,666.7 \text{ k} \cdot \text{ft}^3}{EI} \qquad (1)$$

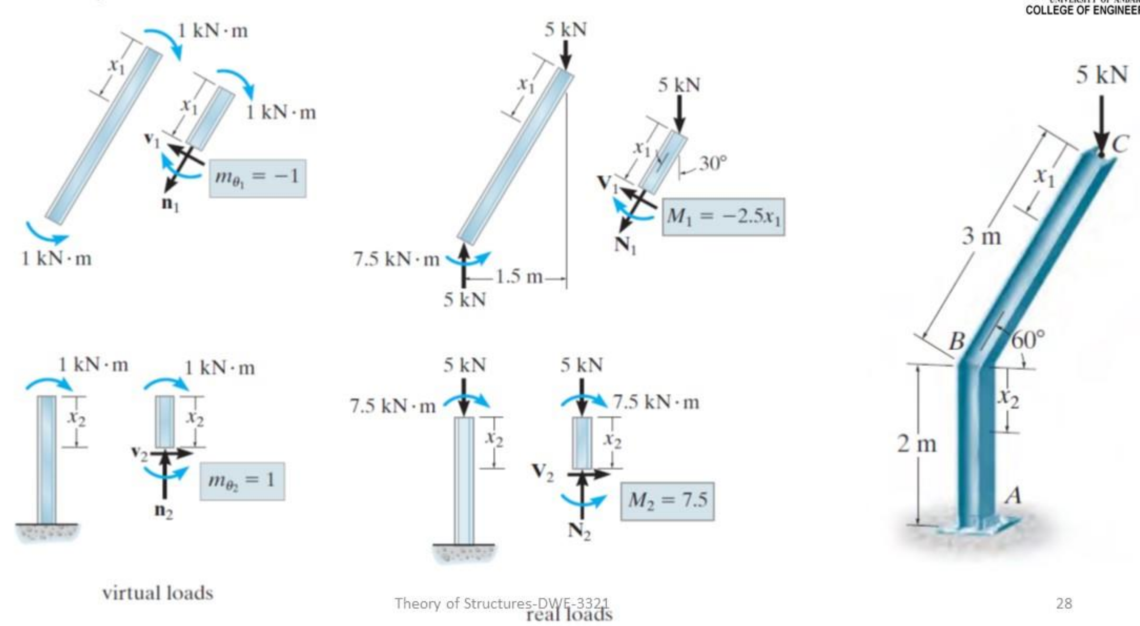
$$\Delta_{C_h} = \frac{13\,666.7 \text{ k} \cdot \text{ft}^3}{[29(10^3) \text{ k/in}^2((12)^2 \text{ in}^2/\text{ft}^2)][600 \text{ in}^4(\text{ft}^4/(12)^4 \text{ in}^4)]}$$

$$= 0.113 \text{ ft} = 1.36 \text{ in.}$$

Ans.

Example: Determine the tangential rotation at point **C** of the frame shown in the figure. Take E = 200 GPa, I = 15x10⁶ mm⁴.

Solution:



$$1 \cdot \theta_C = \int_0^L \frac{m_\theta M}{EI} dx = \int_0^3 \frac{(-1)(-2.5x_1)\, dx_1}{EI} + \int_0^2 \frac{(1)(7.5)\, dx_2}{EI}$$

$$\theta_C = \frac{11.25}{EI} + \frac{15}{EI} = \frac{26.25 \text{ kN} \cdot \text{m}^2}{EI}$$

or

$$\theta_C = \frac{26.25 \text{ kN} \cdot \text{m}^2}{200(10^6) \text{ kN/m}^2[15(10^6) \text{ mm}^4](10^{-12} \text{ m}^4/\text{mm}^4)}$$

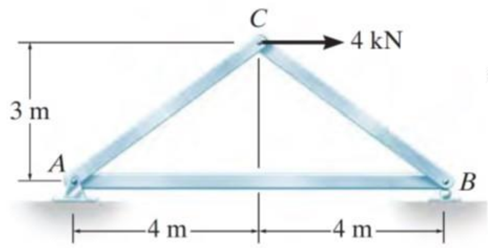
$$= 0.00875 \text{ rad}$$ *Ans.*

Castigliano's Theorem

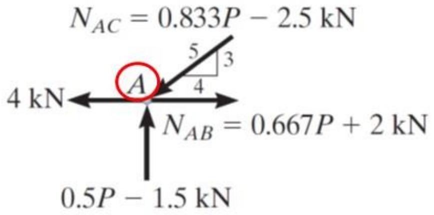
1- Trusses :

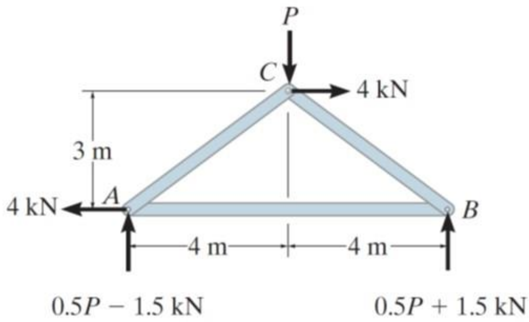
$$\boxed{\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}}$$

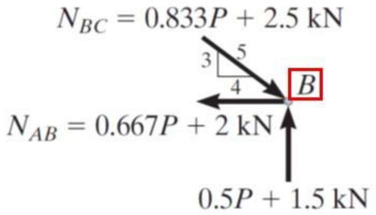
Example: Determine the vertical displacement at Joint **C** of the truss shown in the figure. The cross-sectional area of each member is A = 400 mm² and E = 200 GPa.



Solution:







Member	N	$\dfrac{\partial N}{\partial P}$	$N\,(P=0)$	L	$N\left(\dfrac{\partial N}{\partial P}\right)L$
AB	$0.667P + 2$	0.667	2	8	10.67
AC	$-(0.833P - 2.5)$	−0.833	2.5	5	−10.42
BC	$-(0.833P + 2.5)$	−0.833	−2.5	5	10.42
					$\Sigma = 10.67$ kN·m

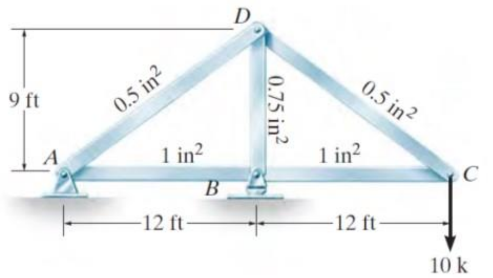
$$\Delta_{C_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{10.67\ \text{kN}\cdot\text{m}}{AE}$$

Substituting $A = 400\ \text{mm}^2 = 400(10^{-6})\ \text{m}^2$, $E = 200\ \text{GPa} = 200(10^9)\ \text{Pa}$, and converting the units of N from kN to N, we have

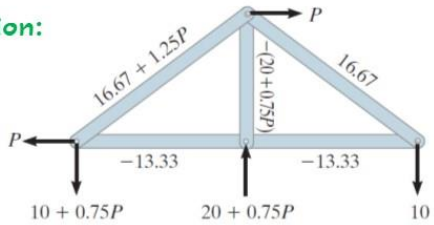
$$\Delta_{C_v} = \frac{10.67(10^3)\ \text{N}\cdot\text{m}}{400(10^{-6})\ \text{m}^2(200(10^9)\ \text{N/m}^2)} = 0.000133\ \text{m} = 0.133\ \text{mm}$$

Ans.

Example: Determine the horizontal displacement at Joint **D** of the truss shown in the figure. The cross-sectional area of each member is indicated in the figure and E = 29(10^3) ksi.



Solution:



Member	N	$\frac{\partial N}{\partial P}$	$N\ (P = 0)$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	-13.33	0	-13.33	12	0
BC	-13.33	0	-13.33	12	0
CD	16.67	0	16.67	15	0
DA	$16.67 + 1.25P$	1.25	16.67	15	312.50
BD	$-(20 + 0.75P)$	-0.75	-20	9	135.00

$$\Delta_{D_h} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = 0 + 0 + 0$$

$$+ \frac{312.50\ \mathrm{k \cdot ft}(12\ \mathrm{in./ft})}{(0.5\ \mathrm{in}^2)[29(10^3)\ \mathrm{k/in}^2]}$$

$$+ \frac{135.00\ \mathrm{k \cdot ft}(12\ \mathrm{in./ft})}{(0.75\ \mathrm{in}^2)[29(10^3)\ \mathrm{k/in}^2]}$$

$$= 0.333\ \mathrm{in.}$$

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Castigliano's Theorem

2- Beams and Frames:

$$\boxed{\Delta = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}}$$

$$\boxed{\theta = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}}$$

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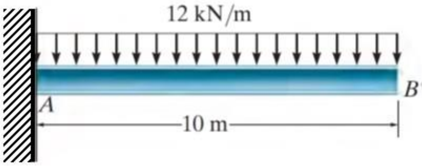
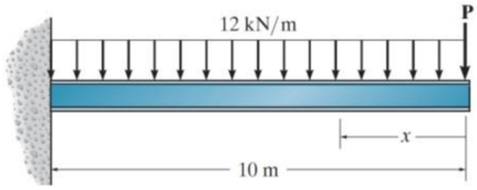
Example: Determine the displacement of point **B** of the steel beam shown in the figure. Take E = 200 GPa, I = 500x10⁶ mm⁴.



Solution:

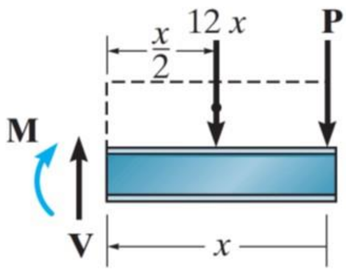


$\circlearrowleft + \Sigma M = 0; \qquad -M - (12x)\left(\dfrac{x}{2}\right) - Px = 0$

$$M = -6x^2 - Px \qquad \frac{\partial M}{\partial P} = -x$$

Setting $P = 0$, its actual value, yields

$$M = -6x^2 \qquad \frac{\partial M}{\partial P} = -x$$



$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{10}\frac{(-6x^2)(-x)\,dx}{EI} = \frac{15(10^3)\text{ kN}\cdot\text{m}^3}{EI}$$

or

$$\Delta_B = \frac{15(10^3)\text{ kN}\cdot\text{m}^3}{200(10^6)\text{ kN/m}^2[500(10^6)\text{ mm}^4](10^{-12}\text{ m}^4/\text{mm}^4)}$$
$$= 0.150\text{ m} = 150\text{ mm} \qquad \textit{Ans.}$$

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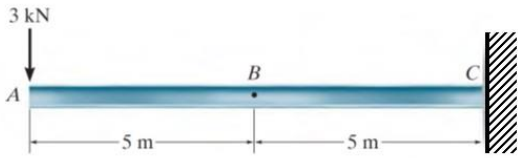
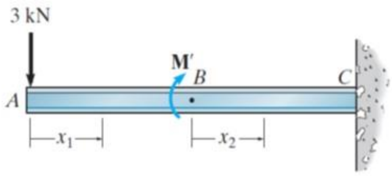
Example: Determine the slop θ at point **B** of the steel beam shown in the figure. Take E = 200 GPa, I = 600x10⁶ mm⁴.



Solution:



(b)

For x_1:

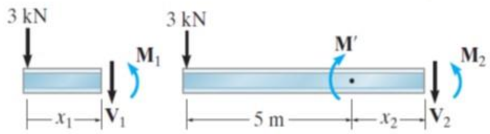
$\circlearrowleft + \Sigma M = 0; \qquad M_1 + 3x_1 = 0$

$$M_1 = -3x_1$$
$$\frac{\partial M_1}{\partial M'} = 0$$

For x_2:

$\circlearrowleft + \Sigma M = 0; \qquad M_2 - M' + 3(5 + x_2) = 0$

$$M_2 = M' - 3(5 + x_2)$$
$$\frac{\partial M_2}{\partial M'} = 1$$



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$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

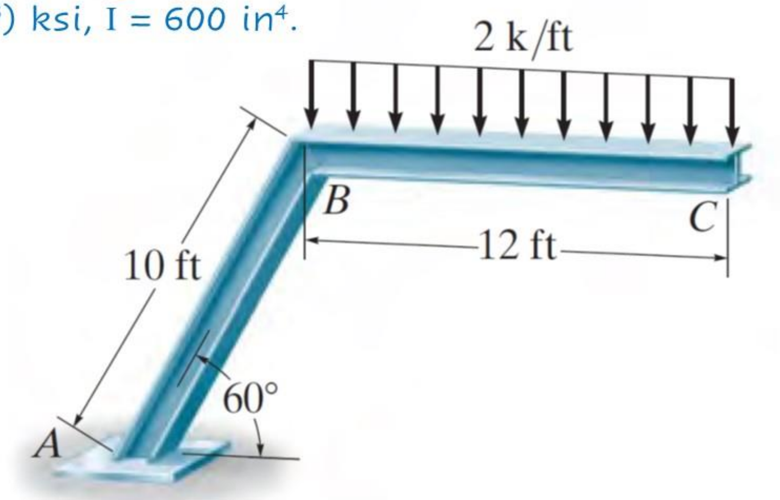
$$= \int_0^5 \frac{(-3x_1)(0)\,dx_1}{EI} + \int_0^5 \frac{-3(5 + x_2)(1)\,dx_2}{EI} = -\frac{112.5\text{ kN}\cdot\text{m}^2}{EI}$$

or

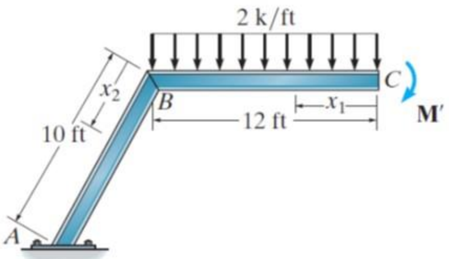
$$\theta_B = \frac{-112.5\text{ kN}\cdot\text{m}^2}{200(10^6)\text{ kN/m}^2[60(10^6)\text{ mm}^4](10^{-12}\text{ m}^4/\text{mm}^4)}$$

$$= -0.00938\text{ rad} \qquad \textit{Ans.}$$

Example: Determine the slop at point **C** of the steel frame shown in the figure. Take $E = 29(10^3)$ ksi, $I = 600$ in^4.



Solution:



For x_1:

$\curvearrowleft + \Sigma M = 0;$ $$-M_1 - 2x_1\left(\frac{x_1}{2}\right) - M' = 0$$

$$M_1 = -\left(x_1^2 + M'\right)$$

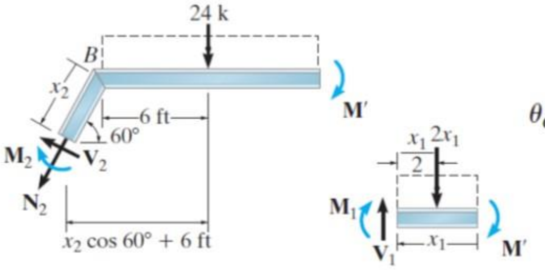
$$\frac{\partial M_1}{\partial M'} = -1$$

For x_2:

$\curvearrowleft + \Sigma M = 0;$ $$-M_2 - 24(x_2 \cos 60^\circ + 6) - M' = 0$$

$$M_2 = -24(x_2 \cos 60^\circ + 6) - M'$$

$$\frac{\partial M_2}{\partial M'} = -1$$



$$\theta_C = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \int_0^{12} \frac{\left(-x_1^2\right)(-1)\,dx_1}{EI} + \int_0^{10} \frac{-24(x_2 \cos 60^\circ + 6)(-1)\,dx_2}{EI}$$

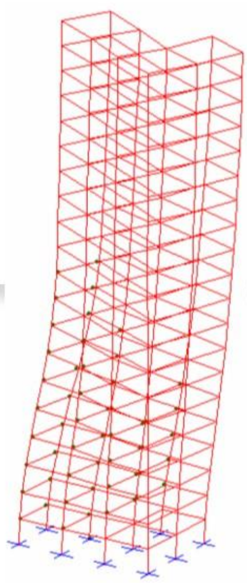
$$= \frac{576\text{ k}\cdot\text{ft}^2}{EI} + \frac{2040\text{ k}\cdot\text{ft}^2}{EI} = \frac{2616\text{ k}\cdot\text{ft}^2}{EI}$$

$$\theta_C = \frac{2616\text{ k}\cdot\text{ft}^2(144\text{ in}^2/\text{ft}^2)}{29(10^3)\text{ k/in}^2(600\text{ in}^4)} = 0.0216\text{ rad}$$ *Ans.*

Unit-7

Analysis of Indeterminate Structures Using Force Methods

Theory of Structures-DWE-3321 1



Indeterminate Structures

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Analysis of Indeterminate Structures

1- Force (Flexibility) Methods: Classical Methods

- Consistent Deformation Method. √
- Castigliano's Second theorem. X

2- Displacement (Stiffness) methods:

- Slope Deflection Method. √
- Moment Distribution Method. √
- Direct Stiffness Method. (maybe)

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Force Method VS. Displacement Methods

	Unknowns	Equations Used for Solution	Coefficients of the Unknowns
Force Method	Forces	Compatibility and Force Displacements	Flexibility Coefficients
Displacement Method	Displacements	Equilibrium and Force Displacement	Stiffness Coefficients

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Consistent Deformation Method:

BEAMS

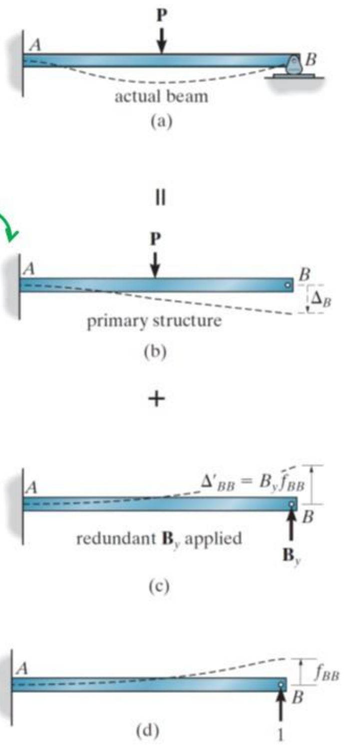
General Analysis Procedure:

$$0 = -\Delta_B + \Delta'_{BB}$$

$$\Delta'_{BB} = B_y f_{BB}$$

$$0 = -\Delta_B + B_y f_{BB}$$

Primary Structures



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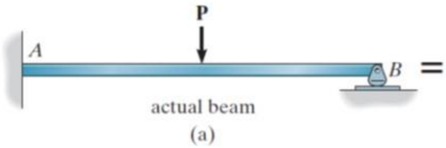
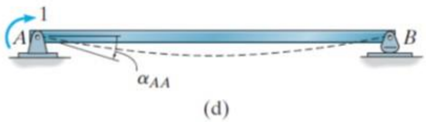
$$\theta'_{AA} = M_A \alpha_{AA}$$



$$0 = \theta_A + M_A \alpha_{AA}$$



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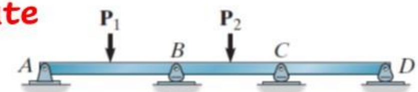
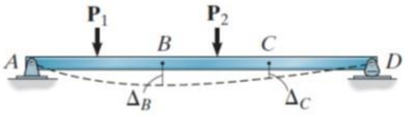
2nd Degree Indeterminate Structure:

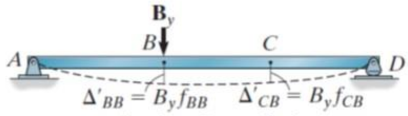


actual beam
(a)



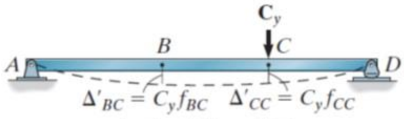
primary structure
(b)

+



redundant $\mathbf{B}_y$ applied
(c)

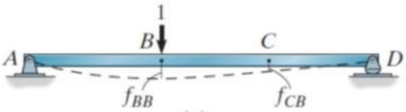
+



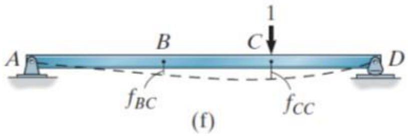
redundant $\mathbf{C}_y$ applied
(d)

$$0 = \Delta_B + B_y f_{BB} + C_y f_{BC}$$

$$0 = \Delta_C + B_y f_{CB} + C_y f_{CC}$$



(e)



(f)

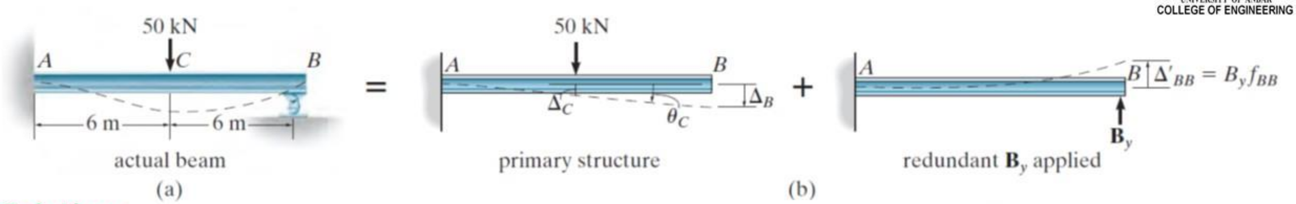
Procedure for Analysis:

Principle of Superposition: Determine the number of degrees n to which the structure is indeterminate. Then specify the n unknown redundant forces or moments that must be removed from the structure in order to make it statically determinate and stable. Using the principle of superposition, draw the statically indeterminate structure and show it to be equal to a series of corresponding statically *determinate* structures.

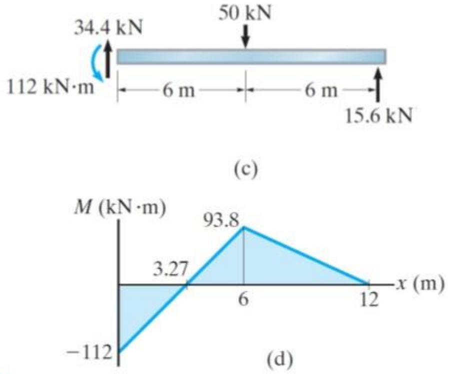
Compatibility Equations: Write a compatibility equation for the displacement or rotation at each point where there is a redundant force or moment. These equations should be expressed in terms of the unknown redundants and their corresponding flexibility coefficients obtained from unit loads or unit couple moments that are collinear with the redundant forces or moments.

Equilibrium Equations: Draw a free-body diagram of the structure. Since the redundant forces and/or moments have been calculated, the remaining unknown reactions can be determined from the equations of equilibrium.

Example : Determine the reaction at the roller support *B* of the beam shown in the figure, *EI* is constant.



Solution :



$$0 = -\Delta_B + B_y f_{BB}$$

$$\Delta_B = \frac{P(L/2)^3}{3EI} + \frac{P(L/2)^2}{2EI}\left(\frac{L}{2}\right)$$

$$= \frac{(50\text{ kN})(6\text{ m})^3}{3EI} + \frac{(50\text{ kN})(6\text{ m})^2}{2EI}(6\text{ m}) = \frac{9000\text{ kN}\cdot\text{m}^3}{EI}\downarrow$$

$$f_{BB} = \frac{PL^3}{3EI} = \frac{1(12\text{ m})^3}{3EI} = \frac{576\text{ m}^3}{EI}\uparrow$$

Substituting these results into Eq. (1) yields

$$(+\uparrow) \qquad 0 = -\frac{9000}{EI} + B_y\left(\frac{576}{EI}\right) \qquad B_y = 15.6\text{ kN} \qquad \textit{Ans.}$$

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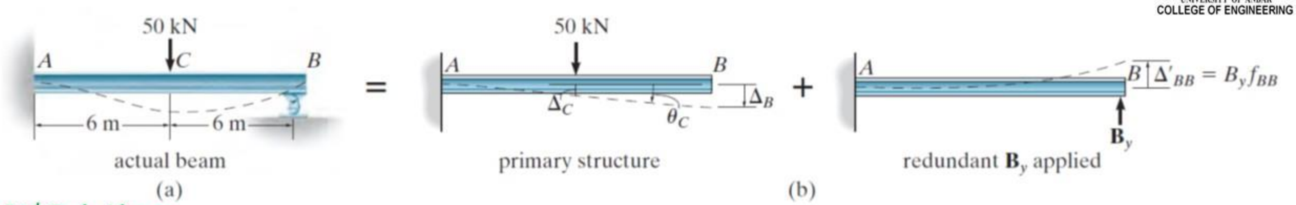
Example : Determine the reaction at the roller support *B* of the beam shown in the figure, *EI* is constant.



2nd Solution :

$$0 = -\Delta_B + B_y f_{BB}$$

$$\Delta_B = \int_0^L \frac{Mm}{EI}dx = \int_0^6 \frac{Mm}{EI}dx + \int_6^{12} \frac{Mm}{EI}dx = \int_0^6 \frac{0.0\times(-x)}{EI}dx + \int_6^{12} \frac{(-50(x-6)\times(-x)}{EI}dx$$

$$\Delta_B = 0.0 + \frac{1}{EI}\int_6^{12}(50x^2 - 300x)dx$$

$$\Delta_B = \frac{1}{EI}\left[\frac{50x^3}{3} - \frac{300x^2}{2}\right]_6^{12} = \frac{9000\ kN.m^3}{EI}\ \downarrow$$

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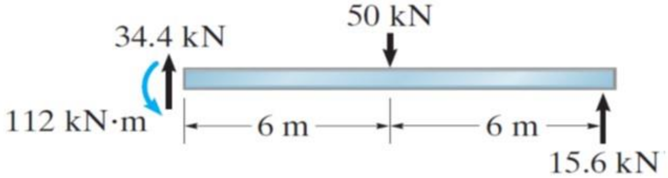
$$f_{BB} = \int_0^L \frac{m.m}{EI} dx$$

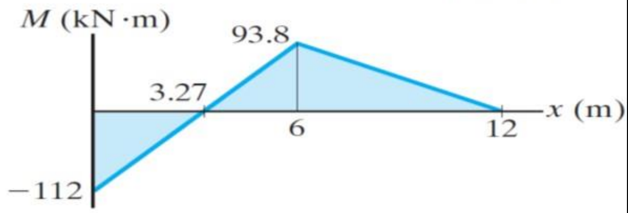
$$f_{BB} = \int_0^{12} \frac{(x)\times(x)}{EI} dx = \int_0^{12} \frac{x^2}{EI} dx$$

$$f_{BB} = \frac{1}{EI} \int_0^{12} \frac{x^3}{3} dx = \frac{1}{EI} \left[\frac{x^3}{3}\right]_0^{12}$$

$$f_{BB} = \frac{1}{EI} \frac{1726}{3} = \frac{576kN.m^3}{EI} \uparrow$$

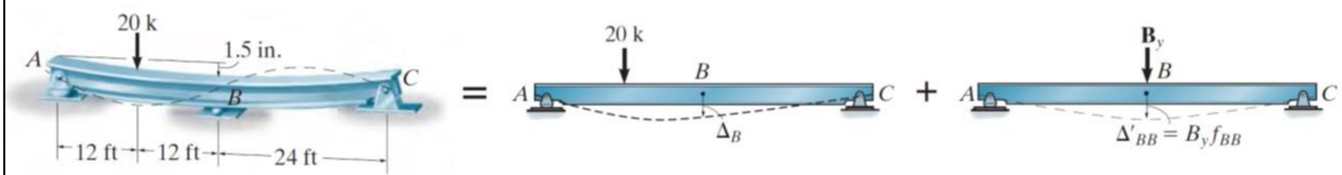


$$0 = -\frac{9000}{EI} + B_y\left(\frac{576}{EI}\right) \quad B_y = 15.6\text{ kN}$$



Example : Draw the shear and moment diagrams for the beam shown in the figure. The support at ***B*** settles 1.5 in. Take E = 29(10^3) ksi and I = 750 in^4.

Solution :



actual beam
(a)

primary structure

redundant $\mathbf{B}_y$ applied

(b)

$$(+\downarrow) \qquad 1.5\text{ in.} = \Delta_B + B_y f_{BB}$$

$$\Delta_B = \frac{Pbx}{6LEI}(L^2 - b^2 - x^2) = \frac{20(12)(24)}{6(48)EI}[(48)^2 - (12)^2 - (24)^2]$$

$$= \frac{31{,}680 \text{ k} \cdot \text{ft}^3}{EI}$$

$$f_{BB} = \frac{PL^3}{48EI} = \frac{1(48)^3}{48EI} = \frac{2304 \text{ k} \cdot \text{ft}^3}{EI}$$

$$1.5 \text{ in.}\,(29(10^3)\text{ k/in}^2)(750 \text{ in}^4)$$

$$= 31{,}680 \text{ k} \cdot \text{ft}^3(12 \text{ in./ft})^3 + B_y(2304 \text{ k} \cdot \text{ft}^3)(12 \text{ in./ft})^3$$

$$B_y = -5.56 \text{ k}$$

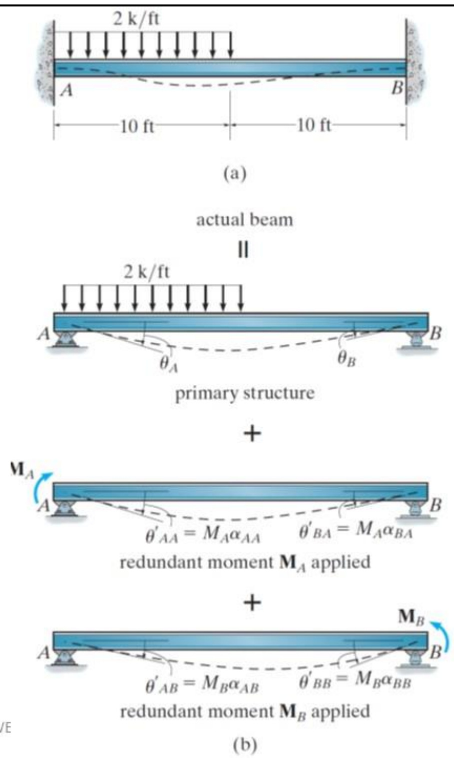
Note : The negative sign indicates that By acts upward on the beam.

Example : Draw the shear and moment diagrams for the beam shown in the figure. *EI* is constant. Neglect the effects of axial load.

Solution :

$$0 = \theta_A + M_A\alpha_{AA} + M_B\alpha_{AB}$$

$$0 = \theta_B + M_A\alpha_{BA} + M_B\alpha_{BB}$$



Using direct equations:
Check Hibbeler

$$\theta_A = \frac{3wL^3}{128EI} = \frac{3(2)(20)^3}{128EI} = \frac{375}{EI}$$

$$\theta_B = \frac{7wL^3}{384EI} = \frac{7(2)(20)^3}{384EI} = \frac{291.7}{EI}$$

$$\alpha_{AA} = \frac{ML}{3EI} = \frac{1(20)}{3EI} = \frac{6.67}{EI}$$

$$\alpha_{BB} = \frac{ML}{3EI} = \frac{1(20)}{3EI} = \frac{6.67}{EI}$$

$$\alpha_{AB} = \frac{ML}{6EI} = \frac{1(20)}{6EI} = \frac{3.33}{EI}$$

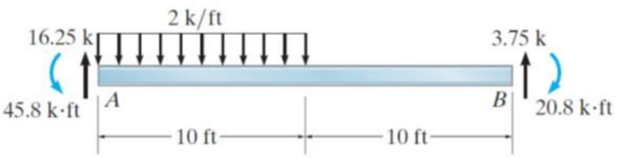
Note that $\alpha_{BA} = \alpha_{AB}$ a consequence of Maxwell's theorem of reciprocal displacements.

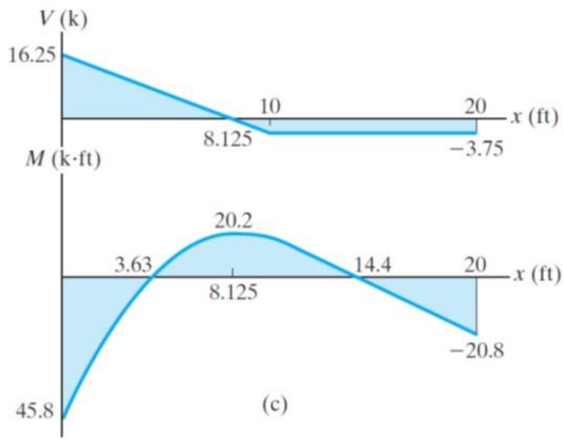
Substitute data in Eqs. (1) and (2):

$$0 = \frac{375}{EI} + M_A\left(\frac{6.67}{EI}\right) + M_B\left(\frac{3.33}{EI}\right)$$

$$0 = \frac{291.7}{EI} + M_A\left(\frac{3.33}{EI}\right) + M_B\left(\frac{6.67}{EI}\right)$$

$$M_A = -45.8 \text{ k}\cdot\text{ft} \qquad M_B = -20.8 \text{ k}\cdot\text{ft}$$

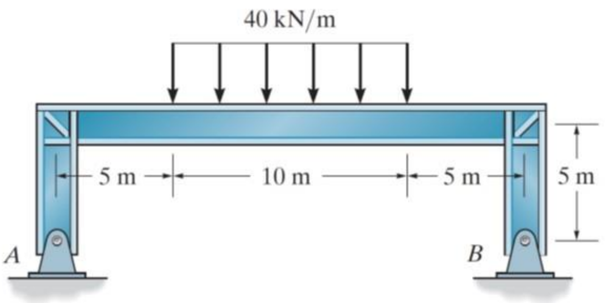




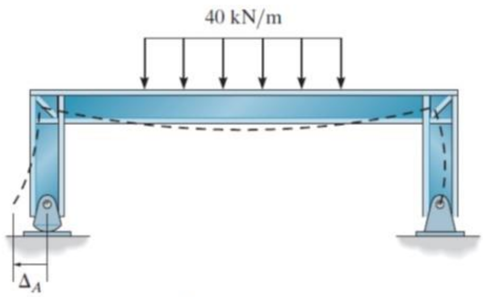
(c)

Consistent Deformation Method : FRAMES

Example : The frame, or bent, shown in the photo is used to support the bridge deck. Assuming EI is constant, Determine the support reactions.

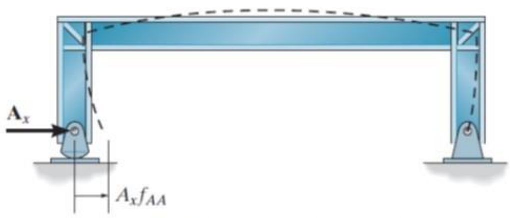


Solution :



Primary structure

+



Redundant force $\mathbf{A}_x$ applied

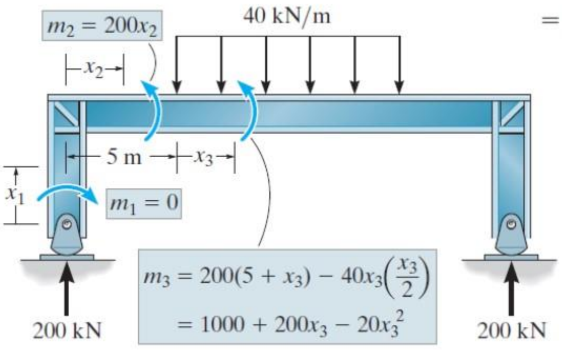
Compatibility Equation :

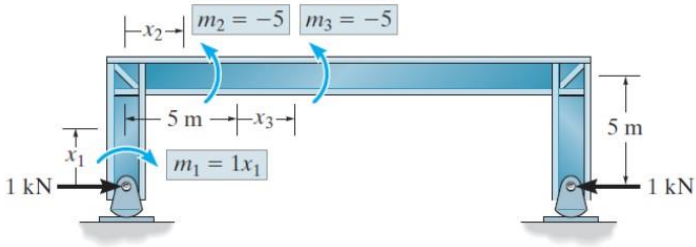
$$(\overset{+}{\rightarrow}) \qquad 0 = \Delta_A + A_x f_{AA}$$

$$\Delta_A = \int_0^L \frac{Mm}{EI}dx = 2\int_0^5 \frac{(0)(1x_1)dx_1}{EI} + 2\int_0^5 \frac{(200x_2)(-5)dx_2}{EI}$$

$$+ 2\int_0^5 \frac{(1000 + 200x_3 - 20x_3^2)(-5)dx_3}{EI}$$

$$= 0 - \frac{25\,000}{EI} - \frac{66\,666.7}{EI} = -\frac{91\,666.7}{EI}$$





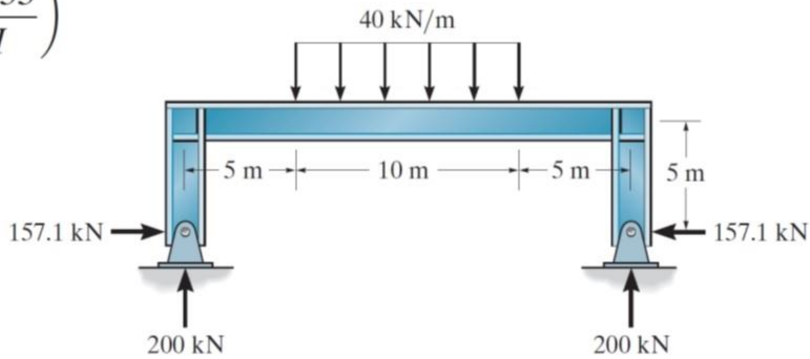
$$f_{AA} = \int_0^L \frac{mm}{EI}dx = 2\int_0^5 \frac{(1x_1)^2 dx_1}{EI} + 2\int_0^5 (5)^2 dx_2 + 2\int_0^5 (5)^2 dx_3$$

$$= \frac{583.33}{EI}$$

$$0 = \frac{-91\,666.7}{EI} + A_x\left(\frac{583.33}{EI}\right)$$

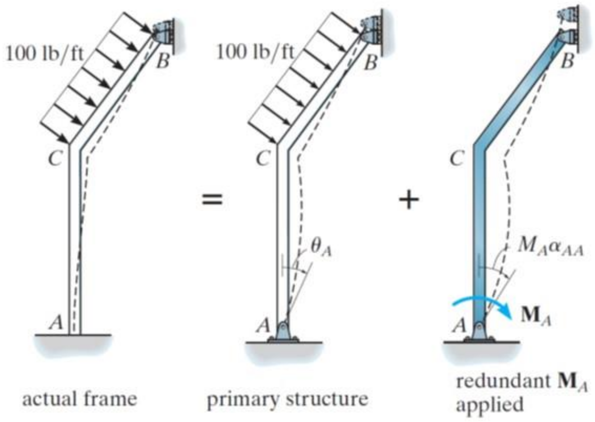
$$A_x = 157 \text{ kN}$$

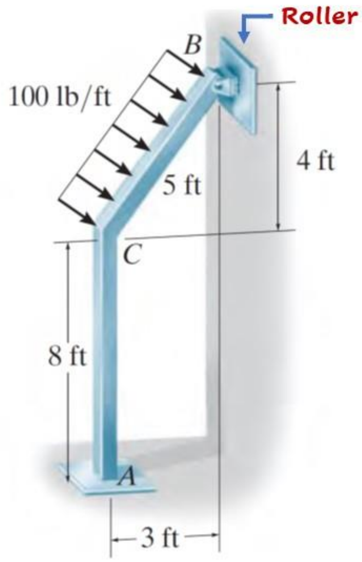
Using Equilibrium Equation $\Rightarrow$



Example : The Determine the moment at the fixed support ***A*** for the frame shown in the figure. EI is constant.

Solution





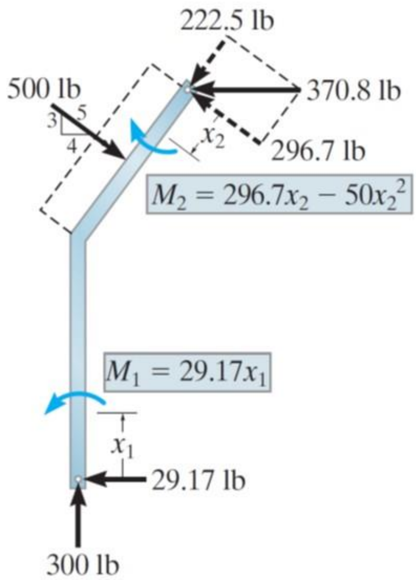
Compatibility Equation : $(\curvearrowleft +) \quad 0 = \theta_A + M_A\alpha_{AA}$

$$\theta_A = \sum \int_0^L \frac{M m_\theta\, dx}{EI}$$

$$= \int_0^8 \frac{(29.17x_1)(1 - 0.0833x_1)\, dx_1}{EI}$$

$$+ \int_0^5 \frac{\left(296.7x_2 - 50x_2^2\right)(0.0667x_2)\, dx_2}{EI}$$

$$= \frac{518.5}{EI} + \frac{303.2}{EI} = \frac{821.8}{EI}$$

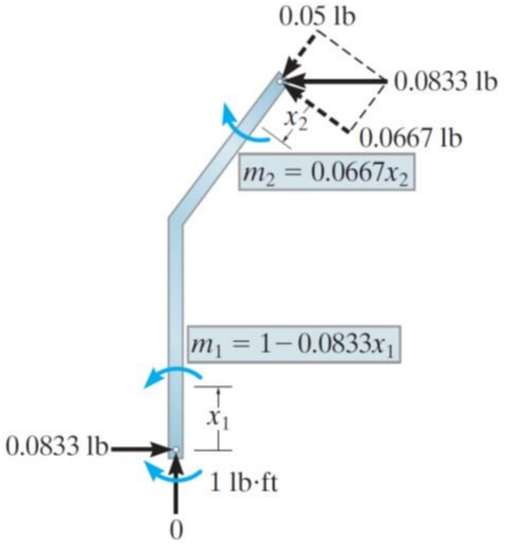


$$\alpha_{AA} = \sum \int_0^L \frac{m_\theta m_\theta}{EI} dx$$

$$= \int_0^8 \frac{(1 - 0.0833x_1)^2\, dx_1}{EI} + \int_0^5 \frac{(0.0667x_2)^2\, dx_2}{EI}$$

$$= \frac{3.85}{EI} + \frac{0.185}{EI} = \frac{4.04}{EI}$$

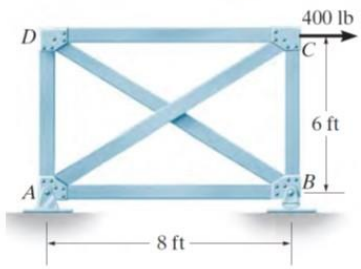
$$0 = \frac{821.8}{EI} + M_A\left(\frac{4.04}{EI}\right) \qquad M_A = -204 \text{ lb} \cdot \text{ft}$$

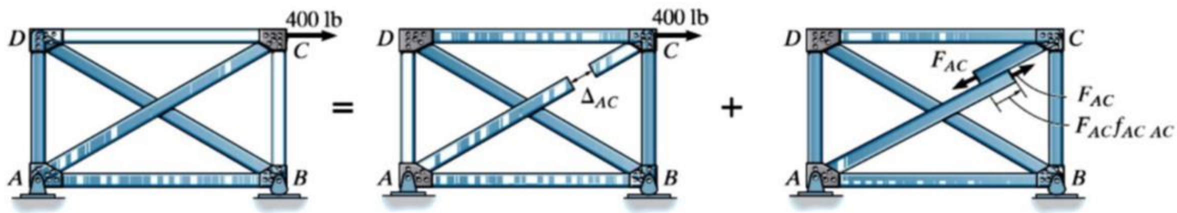


Consistent Deformation Method : TRUSSES

Example : The Determine the force in member AC of the truss shown in the figure. ***AE*** is the same for all the members.

Solution : $0 = \Delta_{AC} + F_{AC} f_{AC\,AC}$





actual truss primary structure redundant $\mathbf{F}_{AC}$ applied

$$\Delta_{AC} = \sum \frac{nNL}{AE}$$

$$= 2\left[\frac{(-0.8)(400)(8)}{AE}\right] + \frac{(-0.6)(0)(6)}{AE} + \frac{(-0.6)(300)(6)}{AE}$$

$$+ \frac{(1)(-500)(10)}{AE} + \frac{(1)(0)(10)}{AE}$$

$$= -\frac{11\,200}{AE}$$

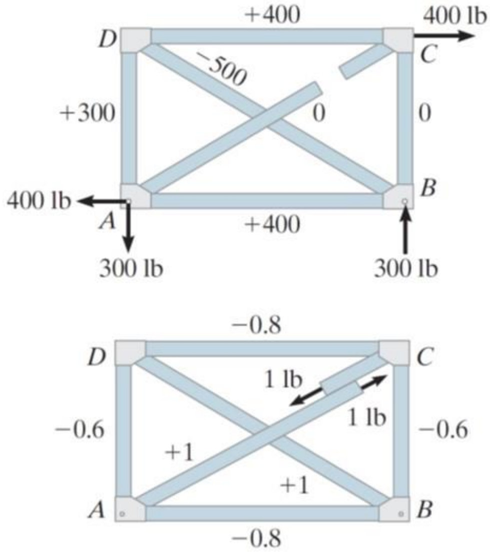
$$f_{AC\,AC} = \sum \frac{n^2L}{AE}$$

$$= 2\left[\frac{(-0.8)^2(8)}{AE}\right] + 2\left[\frac{(-0.6)^2(6)}{AE}\right] + 2\left[\frac{(1)^2 10}{AE}\right]$$

$$= \frac{34.56}{AE}$$

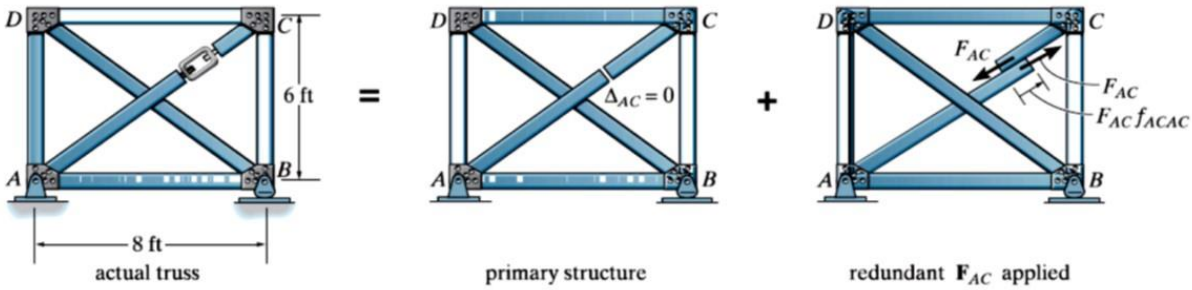
$$0 = -\frac{11\,200}{AE} + \frac{34.56}{AE} F_{AC}$$

$$F_{AC} = 324 \text{ lb (T)}$$



Example : Determine the force in each member of the truss shown in the figure if the turnbuckle on member ***AC*** is used to shorten the member by 0.5 in. Each bar has a cross-sectional area of 0.2 in², and ***E*** = 2911062 psi.

Solution :

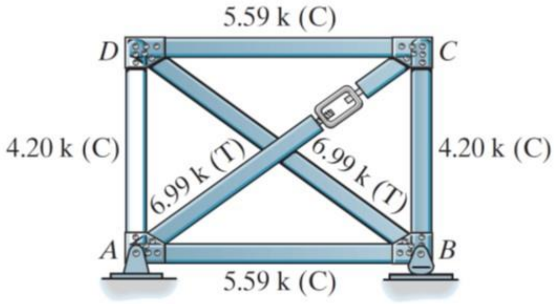


$$f_{AC\,AC} = \frac{34.56}{AE}$$ From previous example

$$0.5 \text{ in.} = 0 + \frac{34.56}{AE} F_{AC}$$

$$0.5 \text{ in.} = 0 + \frac{34.56 \text{ ft}(12 \text{ in./ft})}{(0.2 \text{ in}^2)[29(10^6) \text{ lb/in}^2]} F_{AC}$$

$$F_{AC} = 6993 \text{ lb} = 6.99 \text{ k (T)}$$

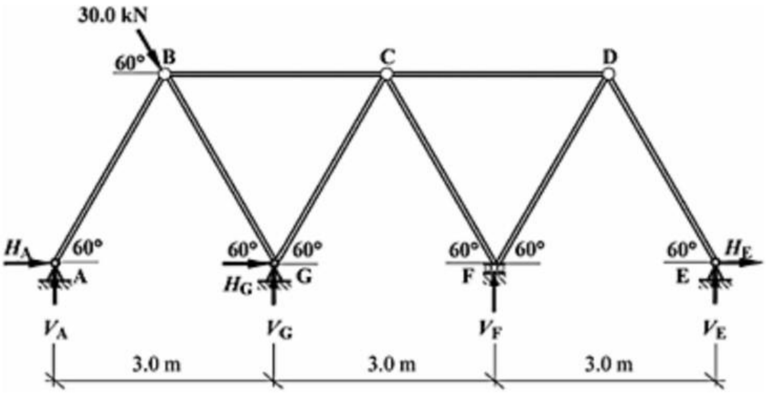


Example : Using the data given, determine the member forces and support reactions for the pinjointed frame shown in the figure. The cross-sectional area of all members is equal to 140 mm². Assume ***E*** = 205 kN/mm².

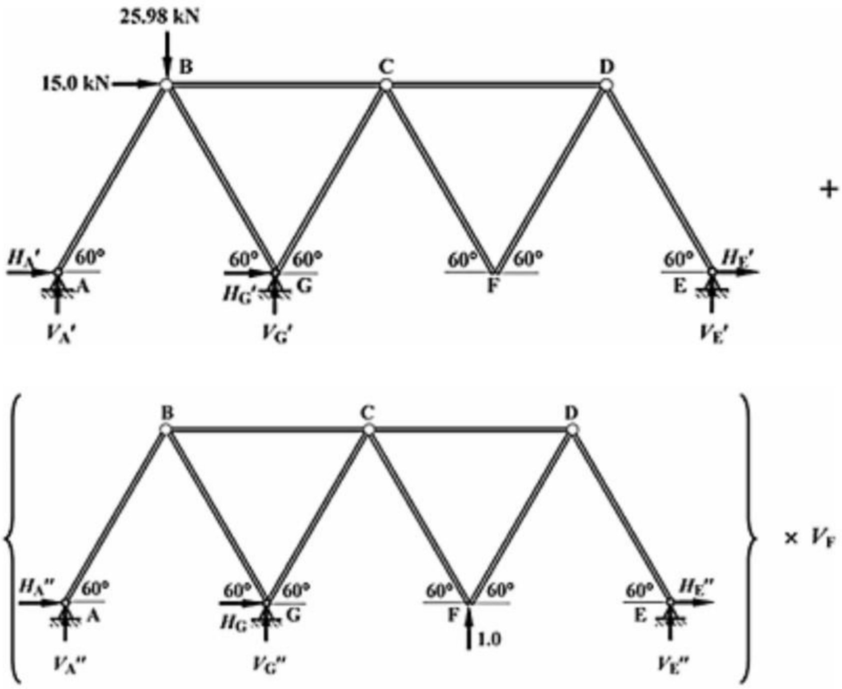
Solution :

All member lengths L=3.0 m
AE = (140×205)= 28.7×10³ kN
Sin60º=0.866, Cos60º=0.5

30 Sin60º = 25.98 kN ↓
30 Cos60º = 15.00 kN →



Compatibility :



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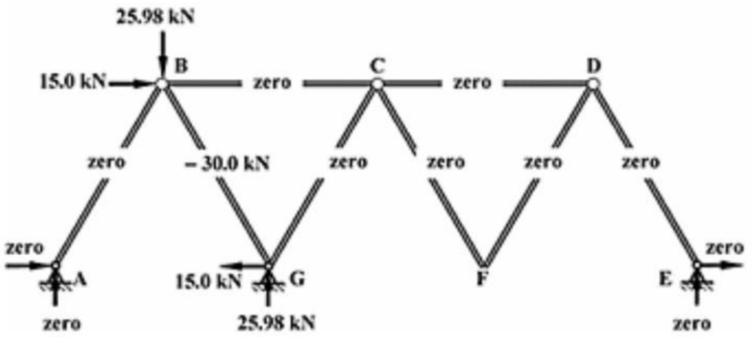
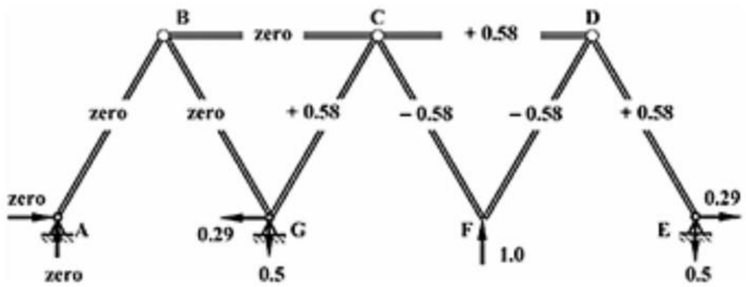
P - forces :



U - forces :



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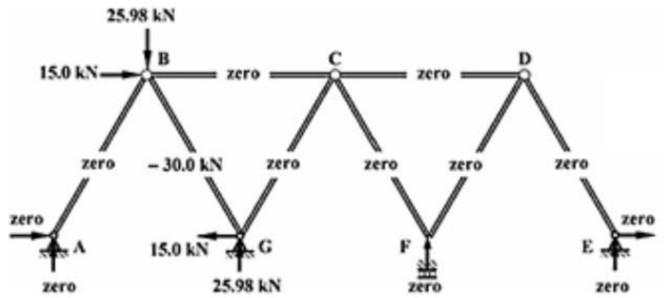
$$\text{i.e. } \sum \frac{PL}{AE}u + \left(\sum \frac{uL}{AE}u\right) \times V_F = 0$$

$$V_F = -\sum \frac{PL}{AE}u \Big/ \sum \frac{uL}{AE}u = 0/0.18 = \text{zero}$$

Final member forces :



Theory of Structures-

Member	Length (mm)	AE (kN)	P-force (kN)	PL/AE (mm)	u	$(PL/AE)\times u$ (mm)	$(uL/AE)\times u$ (mm)
AB	3000	28.7×10^3	0	0	0	0	0
BC	3000	28.7×10^3	0	0	0	0	0
CD	3000	28.7×10^3	0	0	+0.58	0	0.035
DE	3000	28.7×10^3	0	0	+0.58	0	0.035
DF	3000	28.7×10^3	0	0	−0.58	0	0.035
CF	3000	28.7×10^3	0	0	−0.58	0	0.035
CG	3000	28.7×10^3	0	0	+0.58	0	0.035
BG	3000	28.7×10^3	−30.00	−3.14	0	0	0
						Σ=zero	Σ=+0.18

Consistent Deformation Method :

COMPOSITE STRUCTURES

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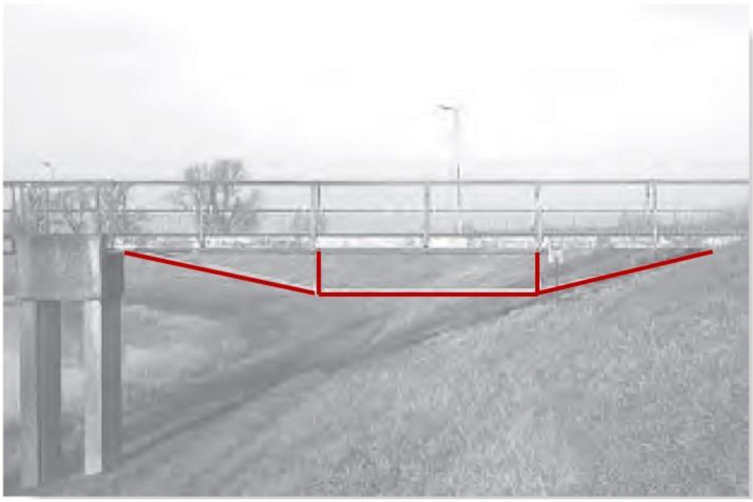

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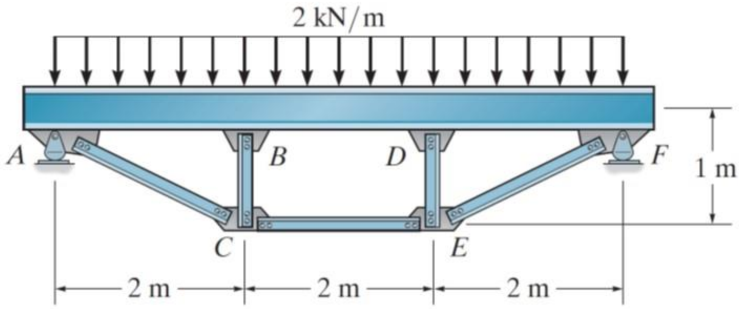
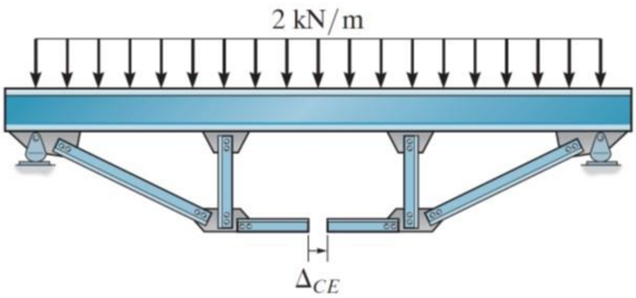
Example : The simply supported queen-post trussed beam shown in the photo is to be designed to support a uniform load of 2 kN/m. The dimensions of the structure are shown in the figure. Determine the force developed in member ***CE***. Neglect the thickness of the beam and assume the truss members are pin connected to the beam. Also, neglect the effect of axial compression and shear in the beam. The cross-sectional area of each strut is 400 mm^2, and for the beam I = 20(10^6) mm^4. Take E = 200 GPa.

Solution :



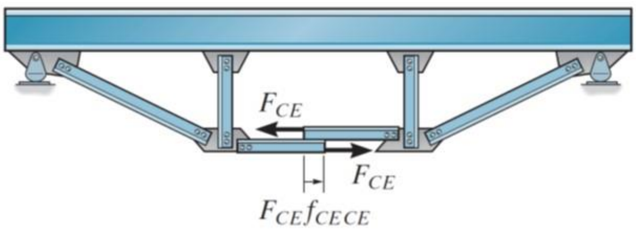
Actual structure



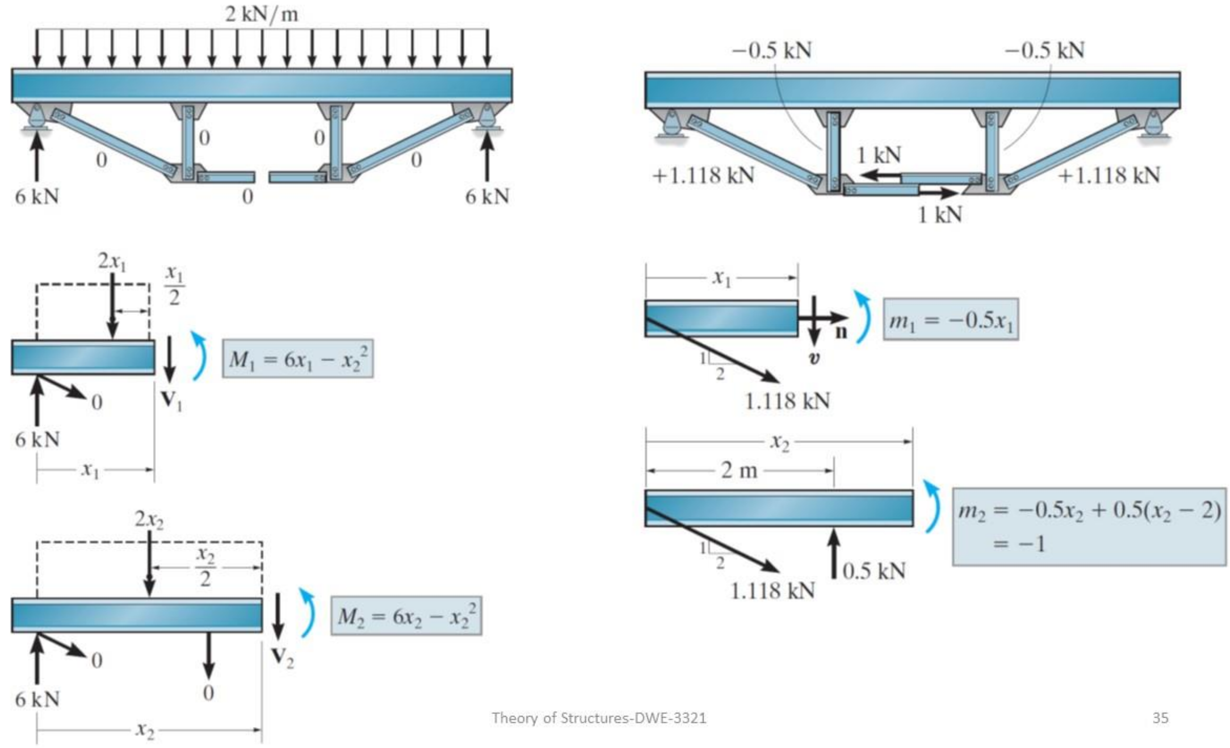
Primary structure

$$0 = \Delta_{CE} + F_{CE} f_{CE\,CE}$$

+



Redundant $\mathbf{F}_{CE}$ applied



$$\Delta_{CE} = \int_0^L \frac{Mm}{EI}dx + \sum\frac{nNL}{AE} = 2\int_0^2 \frac{(6x_1 - x_1^2)(-0.5x_1)dx_1}{EI}$$

$$+ 2\int_2^3 \frac{(6x_2 - x_2^2)(-1)dx_2}{EI} + 2\left(\frac{(1.118)(0)(\sqrt{5})}{AE}\right)$$

$$+ 2\left(\frac{(-0.5)(0)(1)}{AE}\right) + \left(\frac{1(0)2}{AE}\right)$$

$$= -\frac{12}{EI} - \frac{17.33}{EI} + 0 + 0 + 0$$

$$= \frac{-29.33(10^3)}{200(10^9)(20)(10^{-6})} = -7.333(10^{-3})\text{ m}$$

$$f_{CE\,CE} = \int_0^L \frac{m^2 dx}{EI} + \sum \frac{n^2 L}{AE} = 2\int_0^2 \frac{(-0.5x_1)^2 dx_1}{EI} + 2\int_2^3 \frac{(-1)^2 dx_2}{EI}$$

$$+ 2\left(\frac{(1.118)^2(\sqrt{5})}{AE}\right) + 2\left(\frac{(-0.5)^2(1)}{AE}\right) + \left(\frac{(1)^2(2)}{AE}\right)$$

$$= \frac{1.3333}{EI} + \frac{2}{EI} + \frac{5.590}{AE} + \frac{0.5}{AE} + \frac{2}{AE}$$

$$= \frac{3.333(10^3)}{200(10^9)(20)(10^{-6})} + \frac{8.090(10^3)}{400(10^{-6})(200(10^9))}$$

$$= 0.9345(10^{-3})\ \text{m/kN}$$

$$0 = -7.333(10^{-3})\ \text{m} + F_{CE}(0.9345(10^{-3})\ \text{m/kN})$$

$$F_{CE} = 7.85\ \text{kN}$$

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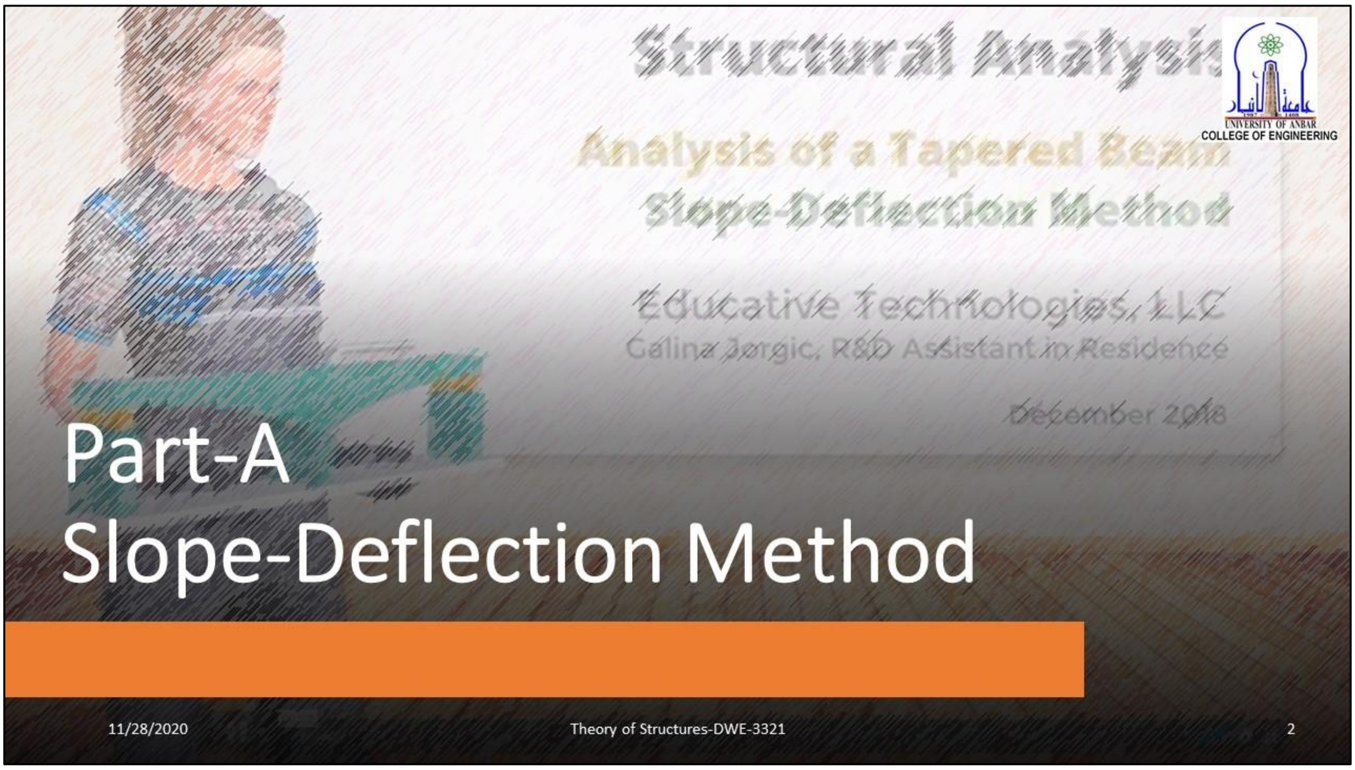
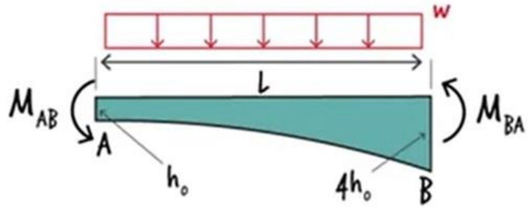
Unit-8

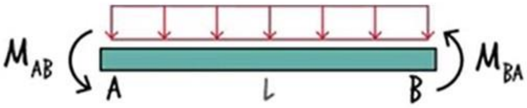
Analysis of Indeterminate Structures
Using Displacement Methods

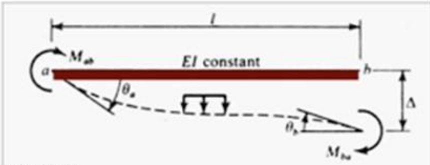
11/28/2020 Theory of Structures-DWE-3321 1

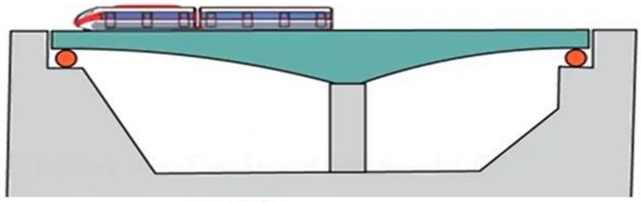


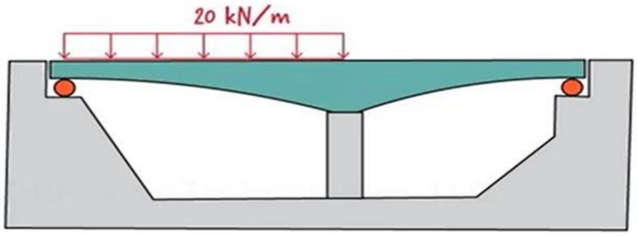
Degrees of Freedom : When a structure is loaded, specified points on it, called *nodes,* will undergo unknown displacements. These displacements are referred to as the *degrees of freedom* for the structure, and in the displacement method of analysis it is important to specify these degrees of freedom since they become the unknowns when the method is applied.











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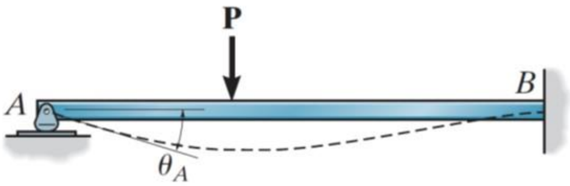
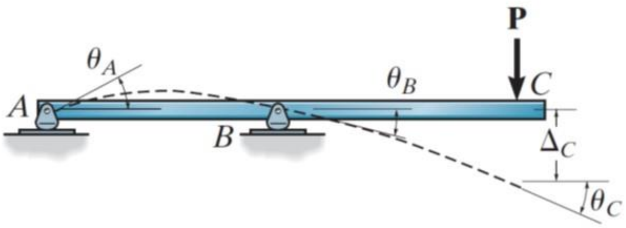
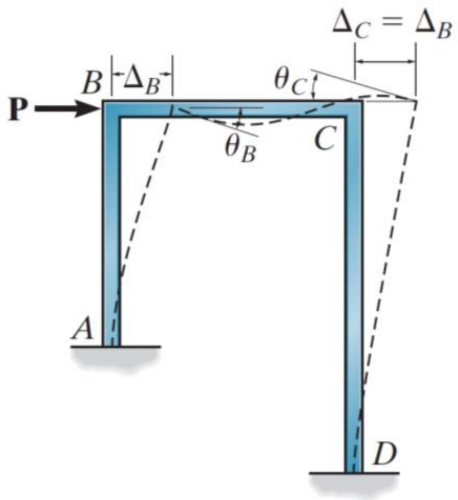
BEAMs & FRAMEs :



(a)

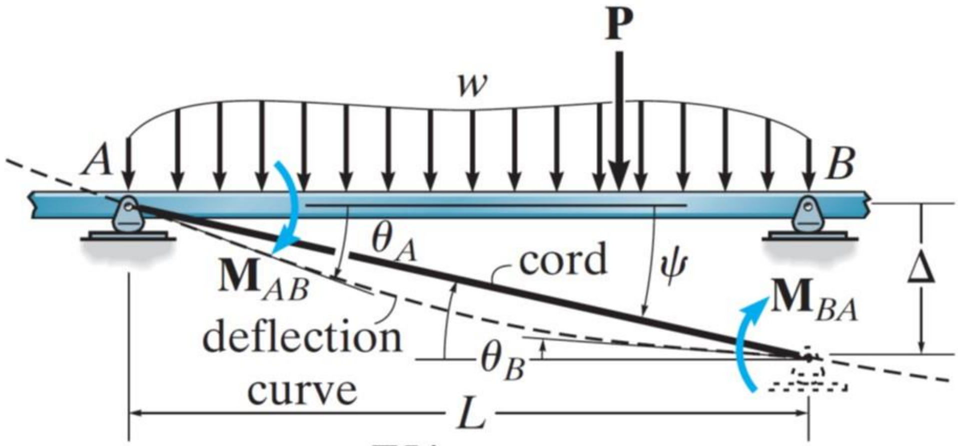


(b)



(c)

General Case :



EI is constant
positive sign convention

Angular Displacement at A, θ_A :

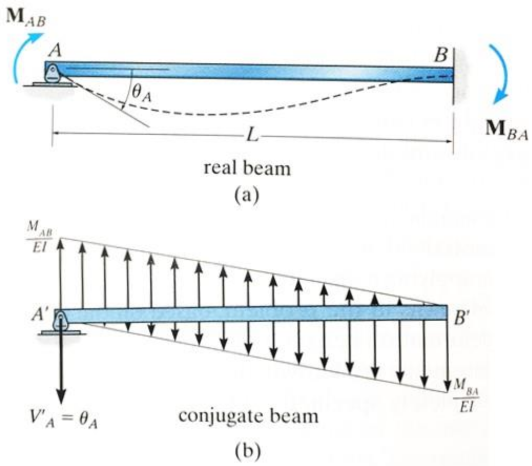
Using Conjugate Beam Method

$$\circlearrowleft + \Sigma M_{A'} = 0; \quad \left[\frac{1}{2}\left(\frac{M_{AB}}{EI}\right)L\right]\frac{L}{3} - \left[\frac{1}{2}\left(\frac{M_{BA}}{EI}\right)L\right]\frac{2L}{3} = 0$$

$$\circlearrowleft + \Sigma M_{B'} = 0; \quad \left[\frac{1}{2}\left(\frac{M_{BA}}{EI}\right)L\right]\frac{L}{3} - \left[\frac{1}{2}\left(\frac{M_{AB}}{EI}\right)L\right]\frac{2L}{3} + \theta_A L = 0$$

$$\boxed{M_{AB} = \frac{4EI}{L}\theta_A}$$

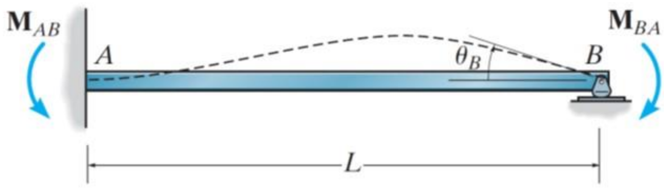
$$\boxed{M_{BA} = \frac{2EI}{L}\theta_A}$$



Angular Displacement at B, θ_B :

$$M_{BA} = \frac{4EI}{L}\theta_B$$

$$M_{AB} = \frac{2EI}{L}\theta_B$$

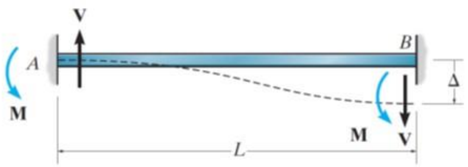


Relative Linear Displacement, Δ:

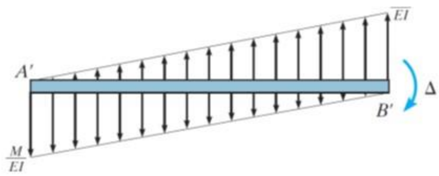
Using Conjugate Beam Method

$$\circlearrowright + \Sigma M_{B'} = 0; \quad \left[\frac{1}{2}\frac{M}{EI}(L)\left(\frac{2}{3}L\right)\right] - \left[\frac{1}{2}\frac{M}{EI}(L)\left(\frac{1}{3}L\right)\right] - \Delta = 0$$

$$M_{AB} = M_{BA} = M = \frac{-6EI}{L^2}\Delta$$



real beam
(a)



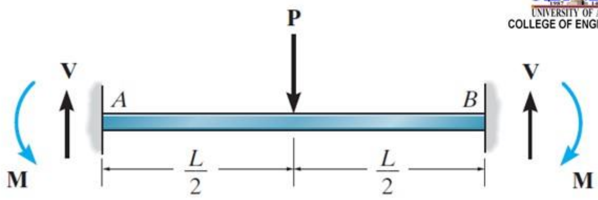
conjugate beam
(b)

Fixed-End Moments:

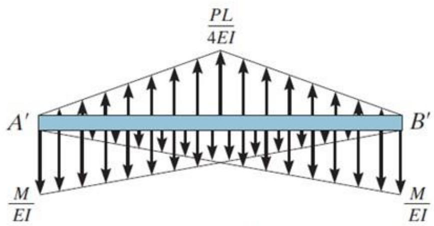
Using Conjugate Beam Method

$$+\uparrow \Sigma F_y = 0; \qquad \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)L\right] - 2\left[\frac{1}{2}\left(\frac{M}{EI}\right)L\right] =$$

$$M = \frac{PL}{8}$$

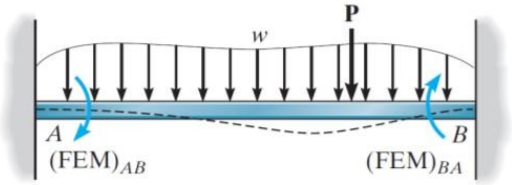


real beam
(a)



conjugate beam
(b)

General Case



$$M_{AB} = (\text{FEM})_{AB} \qquad M_{BA} = (\text{FEM})_{BA}$$

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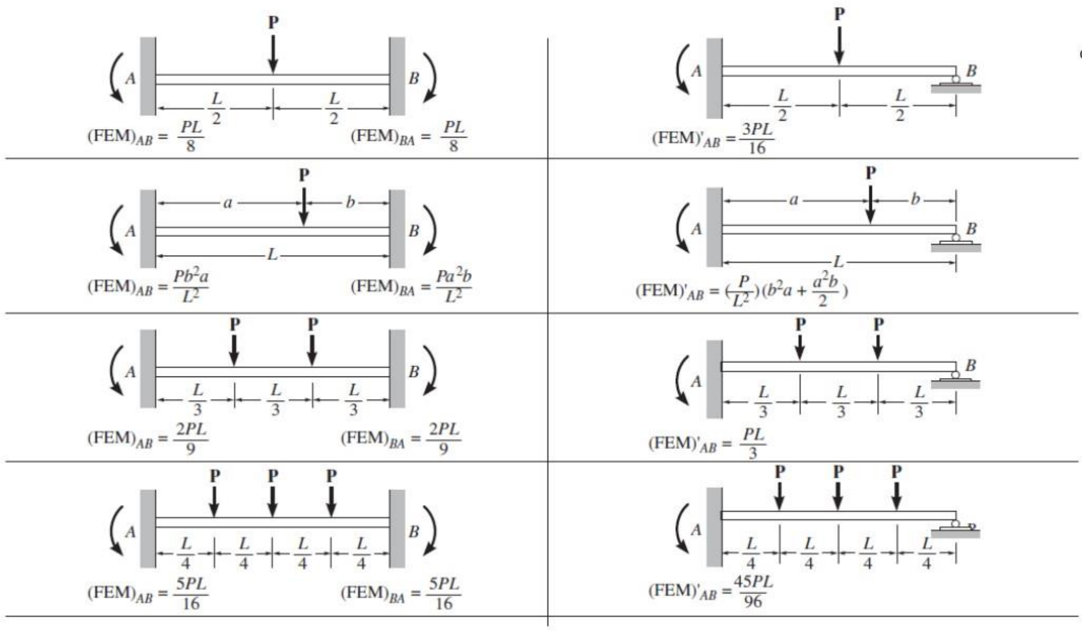
Fixed End Moments



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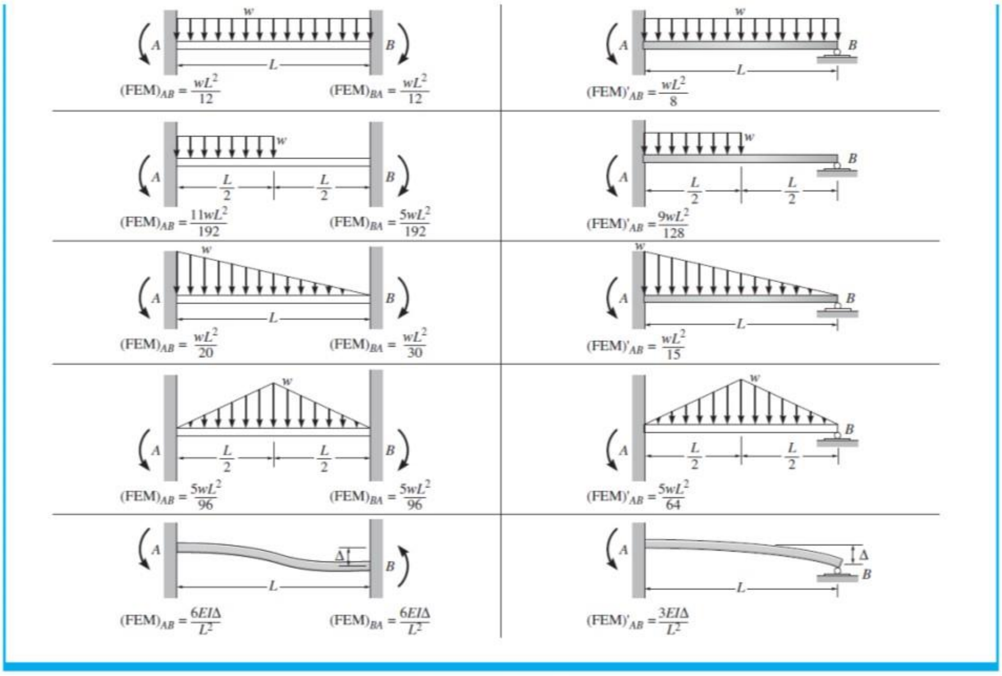


Slope-Deflection Equations :

If we add all the effects of θ_A, θ_B and Δ we get the following slope-deflection equations :

$$M_{AB} = 2E\left(\frac{I}{L}\right)\left[2\theta_A + \theta_B - 3\left(\frac{\Delta}{L}\right)\right] + (\text{FEM})_{AB}$$

$$M_{BA} = 2E\left(\frac{I}{L}\right)\left[2\theta_B + \theta_A - 3\left(\frac{\Delta}{L}\right)\right] + (\text{FEM})_{BA}$$

Since these two equations are similar, the result can be expressed as a single equation. Referring to one end of the span as the near end (**N**) and the other end as the far end (**F**), and letting the member stiffness be represented as $k = I/L$ and the span's cord rotation as $\psi\ (psi) = \Delta/L$ we can write

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For Internal Span or End Span with Far End Fixed

where

M_N = internal moment in the near end of the span; this moment is *positive clockwise* when acting on the span.

E, k = modulus of elasticity of material and span stiffness $k = I/L$.

θ_N, θ_F = near- and far-end slopes or angular displacements of the span at the supports; the angles are measured in *radians* and are *positive clockwise*.

ψ = span rotation of its cord due to a linear displacement, that is, $\psi = \Delta/L$; this angle is measured in *radians* and is *positive clockwise*.

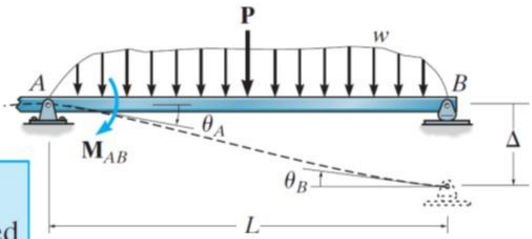
$(\text{FEM})_N$ = fixed-end moment at the near-end support; the moment is *positive clockwise* when acting on the span; refer to the table on the inside back cover for various loading conditions.

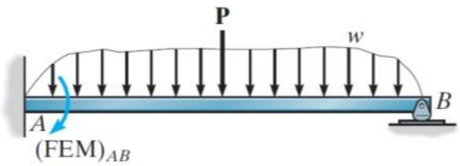
Pin-Supported End Span :

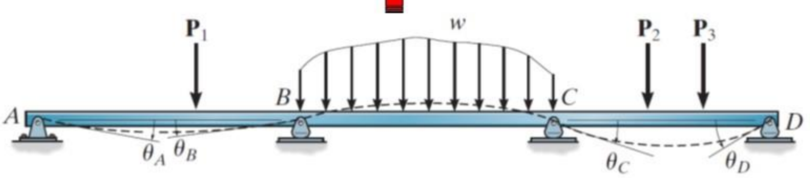
$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$
$$0 = 2Ek(2\theta_F + \theta_N - 3\psi) + 0$$

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

Only for End Span with Far End Pinned or Roller Supported







Procedure for Analysis :

Degrees of Freedom

Slope-Deflection Equations

Equilibrium Equations

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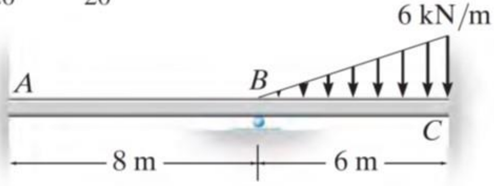
Example : Draw the shear and moment diagrams for the beam shown in the figure. *EI* is constant.

Solution :

Degrees of Freedom **= 1**

$$(\text{FEM})_{BC} = -\frac{wL^2}{30} = -\frac{6(6)^2}{30} = -7.2 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{CB} = \frac{wL^2}{20} = \frac{6(6)^2}{20} = 10.8 \text{ kN}\cdot\text{m}$$



Slope-Deflection Equations

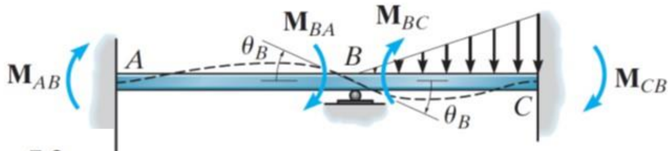
$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{8}\right)[2(0) + \theta_B - 3(0)] + 0 = \frac{EI}{4}\theta_B$$

$$M_{BA} = 2E\left(\frac{I}{8}\right)[2\theta_B + 0 - 3(0)] + 0 = \frac{EI}{2}\theta_B$$

$$M_{BC} = 2E\left(\frac{I}{6}\right)[2\theta_B + 0 - 3(0)] - 7.2 = \frac{2EI}{3}\theta_B - 7.2$$

$$M_{CB} = 2E\left(\frac{I}{6}\right)[2(0) + \theta_B - 3(0)] + 10.8 = \frac{EI}{3}\theta_B + 10.8$$



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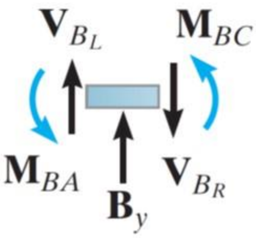
Equilibrium Equations

$\circlearrowright + \Sigma M_B = 0; \qquad M_{BA} + M_{BC} = 0$



$$\frac{EI}{2}\theta_B + \left(\frac{2EI}{3}\theta_B - 7.2\right) = 0 \quad \Longrightarrow \theta_B = \frac{6.17}{EI} \Longrightarrow$$

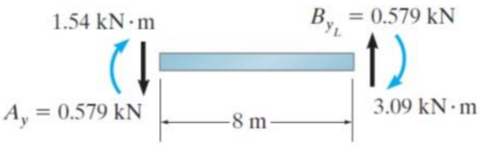
$M_{AB} = 1.54 \text{ kN}\cdot\text{m}$

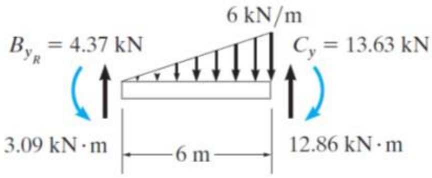
$M_{BA} = 3.09 \text{ kN}\cdot\text{m}$

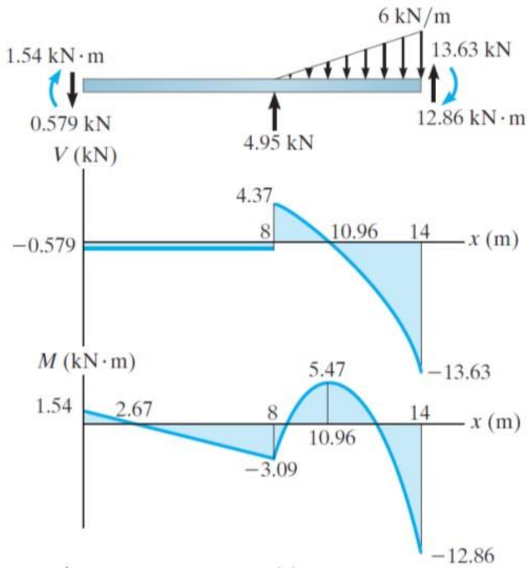
$M_{BC} = -3.09 \text{ kN}\cdot\text{m}$

$M_{CB} = 12.86 \text{ kN}\cdot\text{m}$

Shear and Bending Moment Diagrams :

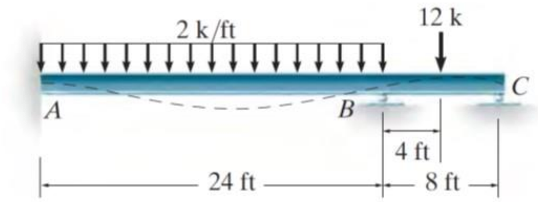






Example : Draw the shear and moment diagrams for the beam shown in the figure. EI is constant.

Solution :



$$(FEM)_{AB} = -\frac{wL^2}{12} = -\frac{1}{12}(2)(24)^2 = -96\ \text{k}\cdot\text{ft}$$

$$(FEM)_{BA} = \frac{wL^2}{12} = \frac{1}{12}(2)(24)^2 = 96\ \text{k}\cdot\text{ft}$$

$$(FEM)_{BC} = -\frac{3PL}{16} = -\frac{3(12)(8)}{16} = -18\ \text{k}\cdot\text{ft}$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{AB} = 2E\left(\frac{I}{24}\right)[2(0) + \theta_B - 3(0)] - 96$$

$$M_{AB} = 0.08333EI\theta_B - 96$$

$$M_{BA} = 2E\left(\frac{I}{24}\right)[2\theta_B + 0 - 3(0)] + 96$$

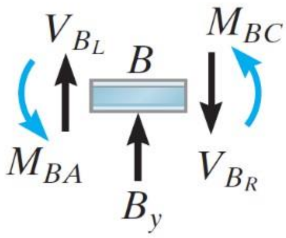
$$M_{BA} = 0.1667EI\theta_B + 96$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (FEM)_N$$

$$M_{BC} = 3E\left(\frac{I}{8}\right)(\theta_B - 0) - 18$$

$$M_{BC} = 0.375EI\theta_B - 18$$

$\circlearrowleft + \Sigma M_B = 0;$ $M_{BA} + M_{BC} = 0$

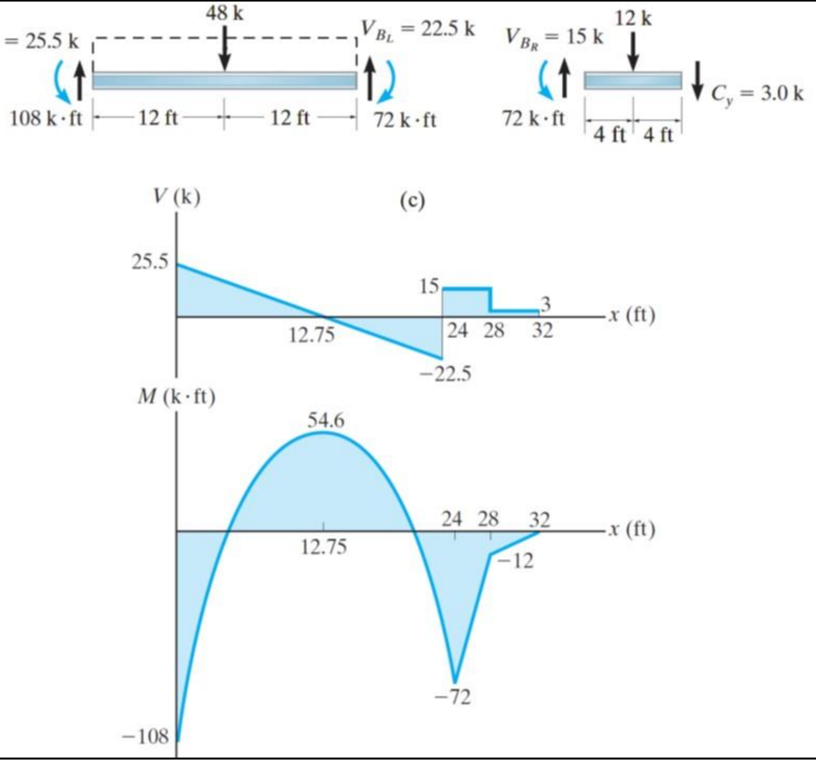


$$\theta_B = -\frac{144.0}{EI}$$

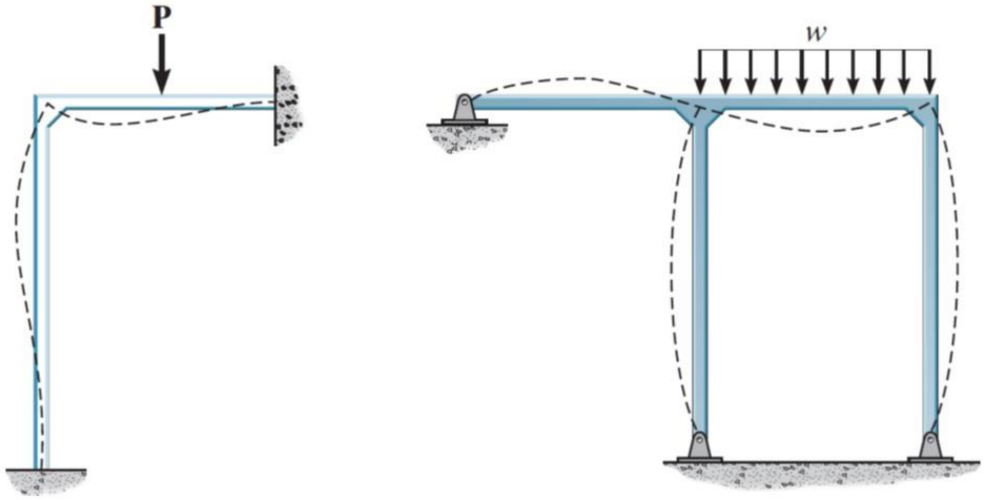
$$M_{AB} = -108.0\ \text{k}\cdot\text{ft}$$

$$M_{BA} = 72.0\ \text{k}\cdot\text{ft}$$

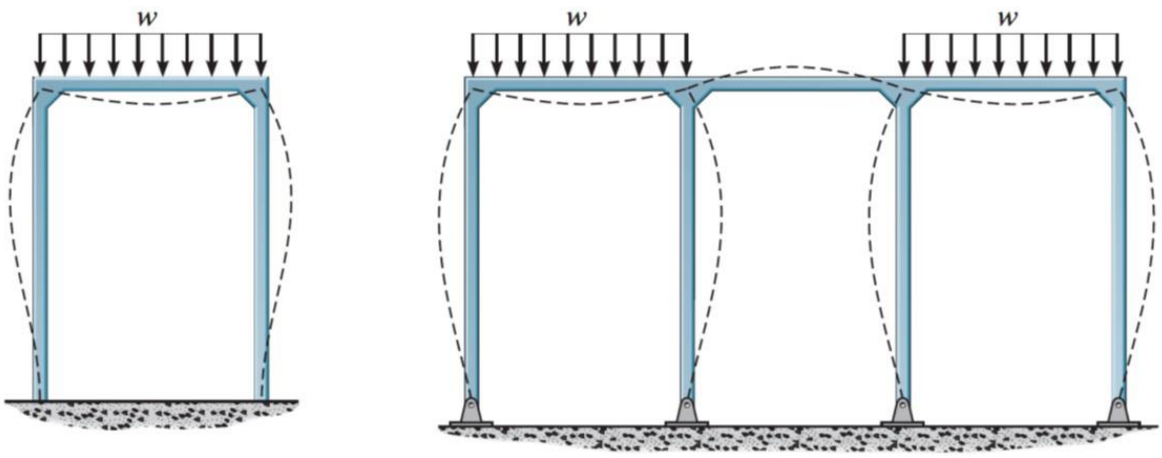
$$M_{BC} = -72.0\ \text{k}\cdot\text{ft}$$



Analysis of FRAMEs – No Sidesway

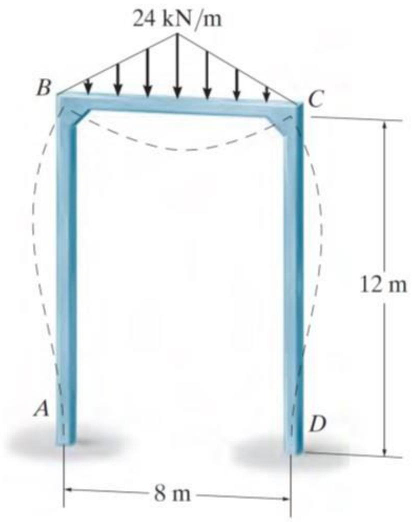


Symmetric Frames :



Example : Draw the shear and moment diagrams for the frame shown in the figure. *EI* is constant.

Solution :



$$(\text{FEM})_{BC} = -\frac{5wL^2}{96} = -\frac{5(24)(8)^2}{96} = -80\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{CB} = \frac{5wL^2}{96} = \frac{5(24)(8)^2}{96} = 80\ \text{kN}\cdot\text{m}$$

Note that $\theta_A = \theta_D = 0$ and $\psi_{AB} = \psi_{BC} = \psi_{CD} = 0$, since no sidesway will occur.

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{12}\right)[2(0) + \theta_B - 3(0)] + 0$$

$$M_{AB} = 0.1667EI\theta_B$$

$$M_{BA} = 2E\left(\frac{I}{12}\right)[2\theta_B + 0 - 3(0)] + 0$$

$$M_{BA} = 0.333EI\theta_B$$

$$M_{BC} = 2E\left(\frac{I}{8}\right)[2\theta_B + \theta_C - 3(0)] - 80$$

$$M_{BC} = 0.5EI\theta_B + 0.25EI\theta_C - 80$$

$$M_{CB} = 2E\left(\frac{I}{8}\right)[2\theta_C + \theta_B - 3(0)] + 80$$

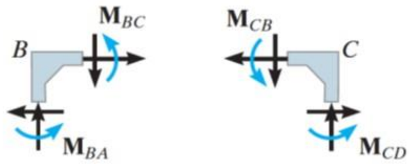
$$M_{CB} = 0.5EI\theta_C + 0.25EI\theta_B + 80$$

$$M_{CD} = 2E\left(\frac{I}{12}\right)[2\theta_C + 0 - 3(0)] + 0$$

$$M_{CD} = 0.333EI\theta_C$$

$$M_{DC} = 2E\left(\frac{I}{12}\right)[2(0) + \theta_C - 3(0)] + 0$$

$$M_{DC} = 0.1667EI\theta_C$$



$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$

$$0.833EI\theta_B + 0.25EI\theta_C = 80$$
$$0.833EI\theta_C + 0.25EI\theta_B = -80$$

$$\theta_B = -\theta_C = \frac{137.1}{EI}$$

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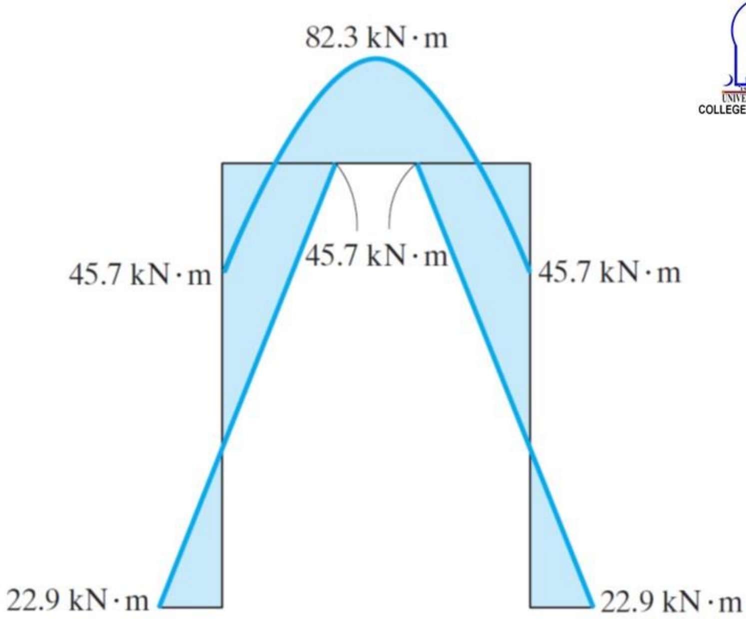
$$M_{BA} = 45.7 \text{ kN}\cdot\text{m}$$
$$M_{BC} = -45.7 \text{ kN}\cdot\text{m}$$
$$M_{CB} = 45.7 \text{ kN}\cdot\text{m}$$
$$M_{CD} = -45.7 \text{ kN}\cdot\text{m}$$
$$M_{DC} = -22.9 \text{ kN}\cdot\text{m}$$



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Example : Determine the internal moments at each joint of the frame shown in the figure. The moment of inertia for each member is given in the figure. Take E = 29(10^3) ksi.

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Solution :

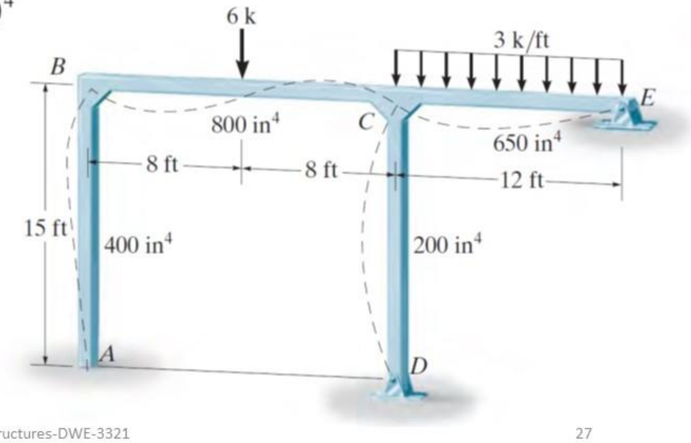
$$k_{AB} = \frac{400}{15(12)^4} = 0.001286\ \text{ft}^3 \qquad k_{CD} = \frac{200}{15(12)^4} = 0.000643\ \text{ft}^3$$

$$k_{BC} = \frac{800}{16(12)^4} = 0.002411\ \text{ft}^3 \qquad k_{CE} = \frac{650}{12(12)^4} = 0.002612\ \text{ft}^3$$

$$(\text{FEM})_{BC} = -\frac{PL}{8} = -\frac{6(16)}{8} = -12\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = \frac{PL}{8} = \frac{6(16)}{8} = 12\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{CE} = -\frac{wL^2}{8} = -\frac{3(12)^2}{8} = -54\ \text{k}\cdot\text{ft}$$



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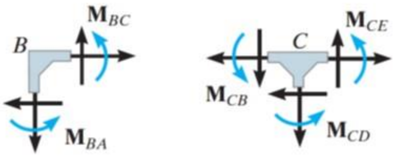
$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$
$$M_{AB} = 2[29(10^3)(12)^2](0.001286)[2(0) + \theta_B - 3(0)] + 0$$
$$M_{AB} = 10740.7\theta_B$$
$$M_{BA} = 2[29(10^3)(12)^2](0.001286)[2\theta_B + 0 - 3(0)] + 0$$
$$M_{BA} = 21\,481.5\theta_B$$
$$M_{BC} = 2[29(10^3)(12)^2](0.002411)[2\theta_B + \theta_C - 3(0)] - 12$$
$$M_{BC} = 40\,277.8\theta_B + 20\,138.9\theta_C - 12$$
$$M_{CB} = 2[29(10^3)(12)^2](0.002411)[2\theta_C + \theta_B - 3(0)] + 12$$
$$M_{CB} = 20\,138.9\theta_B + 40\,277.8\theta_C + 12$$
$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$
$$M_{CD} = 3[29(10^3)(12)^2](0.000643)[\theta_C - 0] + 0$$
$$M_{CD} = 8055.6\theta_C$$
$$M_{CE} = 3[29(10^3)(12)^2](0.002612)[\theta_C - 0] - 54$$
$$M_{CE} = 32\,725.7\theta_C - 54$$



$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} + M_{CE} = 0$$

$$61\,759.3\theta_B + 20\,138.9\theta_C = 12$$
$$20\,138.9\theta_B + 81\,059.0\theta_C = 42$$

$$\theta_B = 2.758(10^{-5}) \text{ rad}$$

$$\theta_C = 5.113(10^{-4}) \text{ rad}$$

$$M_{AB} = 0.296 \text{ k} \cdot \text{ft}$$
$$M_{BA} = 0.592 \text{ k} \cdot \text{ft}$$
$$M_{BC} = -0.592 \text{ k} \cdot \text{ft}$$
$$M_{CB} = 33.1 \text{ k} \cdot \text{ft}$$
$$M_{CD} = 4.12 \text{ k} \cdot \text{ft}$$
$$M_{CE} = -37.3 \text{ k} \cdot \text{ft}$$

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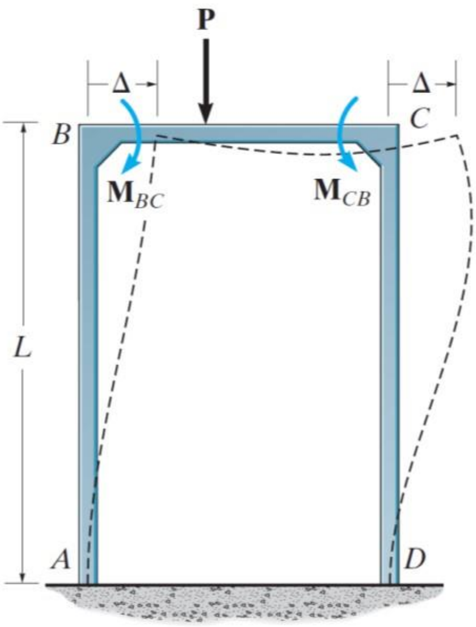
Analysis of FRAMEs – Sidesway



Example : Determine the moments at each joint of the frame shown in the figure. EI is constant.

Solution :

$$M_{AB} = 2E\left(\frac{I}{12}\right)\left[2(0) + \theta_B - 3\left(\frac{18}{12}\psi_{DC}\right)\right] + 0 = EI(0.1667\theta_B - 0.75\psi_{DC})$$

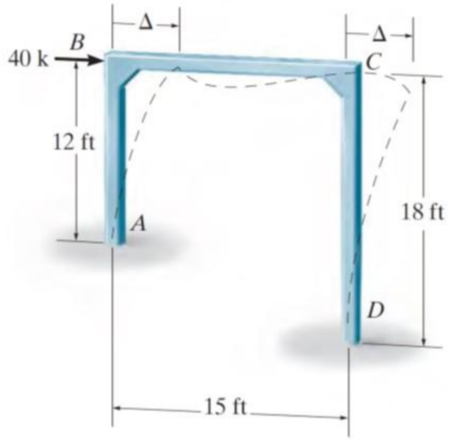
$$M_{BA} = 2E\left(\frac{I}{12}\right)\left[2\theta_B + 0 - 3\left(\frac{18}{12}\psi_{DC}\right)\right] + 0 = EI(0.3333\theta_B - 0.75\psi_{DC})$$

$$M_{BC} = 2E\left(\frac{I}{15}\right)[2\theta_B + \theta_C - 3(0)] + 0 = EI(0.267\theta_B + 0.133\theta_C)$$

$$M_{CB} = 2E\left(\frac{I}{15}\right)[2\theta_C + \theta_B - 3(0)] + 0 = EI(0.267\theta_C + 0.133\theta_B)$$

$$M_{CD} = 2E\left(\frac{I}{18}\right)[2\theta_C + 0 - 3\psi_{DC}] + 0 = EI(0.222\theta_C - 0.333\psi_{DC})$$

$$M_{DC} = 2E\left(\frac{I}{18}\right)[2(0) + \theta_C - 3\psi_{DC}] + 0 = EI(0.111\theta_C - 0.333\psi_{DC})$$



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$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 40 - V_A - V_D = 0$$

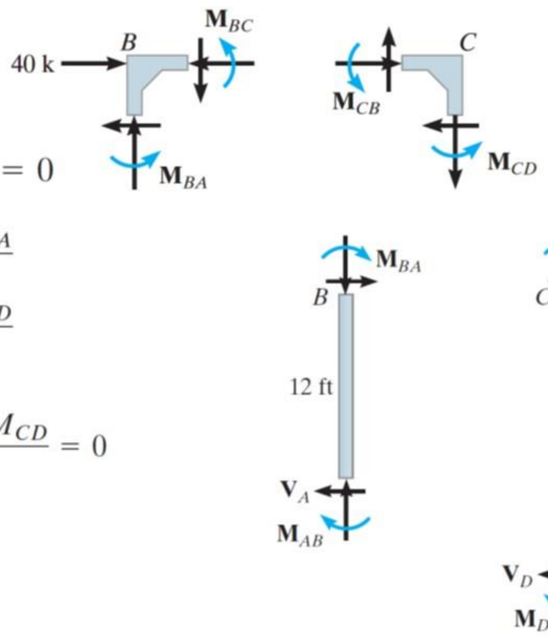
$$\Sigma M_B = 0; \qquad V_A = -\frac{M_{AB} + M_{BA}}{12}$$

$$\Sigma M_C = 0; \qquad V_D = -\frac{M_{DC} + M_{CD}}{18}$$

Thus,

$$40 + \frac{M_{AB} + M_{BA}}{12} + \frac{M_{DC} + M_{CD}}{18} = 0$$

$$0.6\theta_B + 0.133\theta_C - 0.75\psi_{DC} = 0$$
$$0.133\theta_B + 0.489\theta_C - 0.333\psi_{DC} = 0$$
$$0.5\theta_B + 0.222\theta_C - 1.944\psi_{DC} = -\frac{480}{EI}$$



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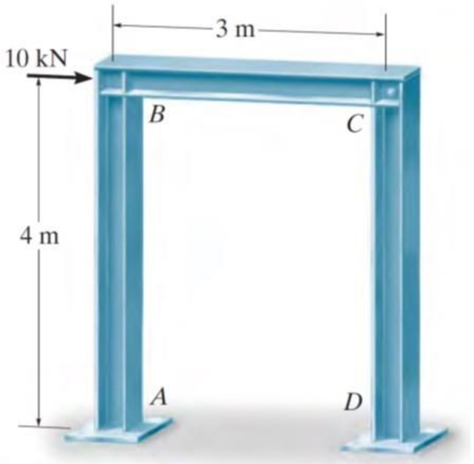
$$EI\theta_B = 438.81 \qquad EI\theta_C = 136.18 \qquad EI\psi_{DC} = 375.26$$

$$M_{AB} = -208 \text{ k} \cdot \text{ft}$$
$$M_{BA} = -135 \text{ k} \cdot \text{ft}$$
$$M_{BC} = 135 \text{ k} \cdot \text{ft}$$
$$M_{CB} = 94.8 \text{ k} \cdot \text{ft}$$
$$M_{CD} = -94.8 \text{ k} \cdot \text{ft}$$
$$M_{DC} = -110 \text{ k} \cdot \text{ft}$$

Example : Determine the moments at each joint of the frame shown in the figure. The supports at **A** and **D** are fixed and joint **C** is assumed pin connected. EI is constant for each member.

Solution :



Slope-Deflection Equations :

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

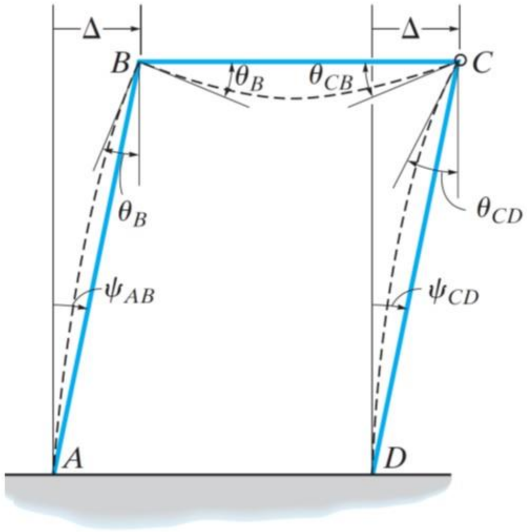
$$M_{AB} = 2E\left(\frac{I}{4}\right)[2(0) + \theta_B - 3\psi] + 0$$

$$M_{BA} = 2E\left(\frac{I}{4}\right)(2\theta_B + 0 - 3\psi) + 0$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BC} = 3E\left(\frac{I}{3}\right)(\theta_B - 0) + 0$$

$$M_{DC} = 3E\left(\frac{I}{4}\right)(0 - \psi) + 0$$



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Equilibrium Equations :

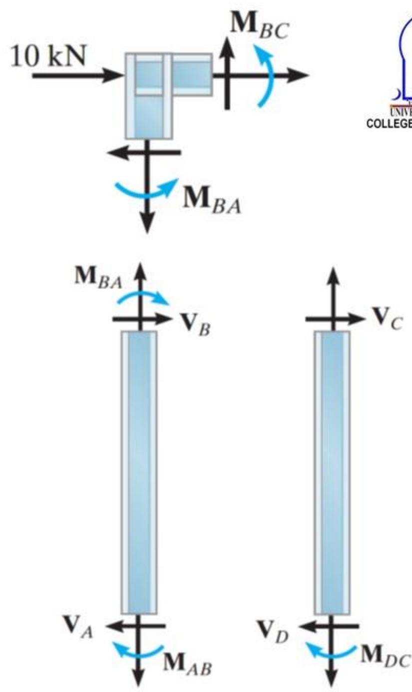
$$M_{BA} + M_{BC} = 0$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 10 - V_A - V_D = 0$$

$$\Sigma M_B = 0; \qquad V_A = -\frac{M_{AB} + M_{BA}}{4}$$

$$\Sigma M_C = 0; \qquad V_D = -\frac{M_{DC}}{4}$$

$$10 + \frac{M_{AB} + M_{BA}}{4} + \frac{M_{DC}}{4} = 0$$



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Substituting :

$$\theta_B = \frac{3}{4}\psi$$

$$10 + \frac{EI}{4}\left(\frac{3}{2}\theta_B - \frac{15}{4}\psi\right) = 0 \quad \Rightarrow \quad \theta_B = \frac{240}{21EI} \qquad \psi = \frac{320}{21EI}$$

$$M_{AB} = -17.1\ \text{kN}\cdot\text{m}, \quad M_{BA} = -11.4\ \text{kN}\cdot\text{m}$$

$$M_{BC} = 11.4\ \text{kN}\cdot\text{m}, \quad M_{DC} = -11.4\ \text{kN}\cdot\text{m}$$

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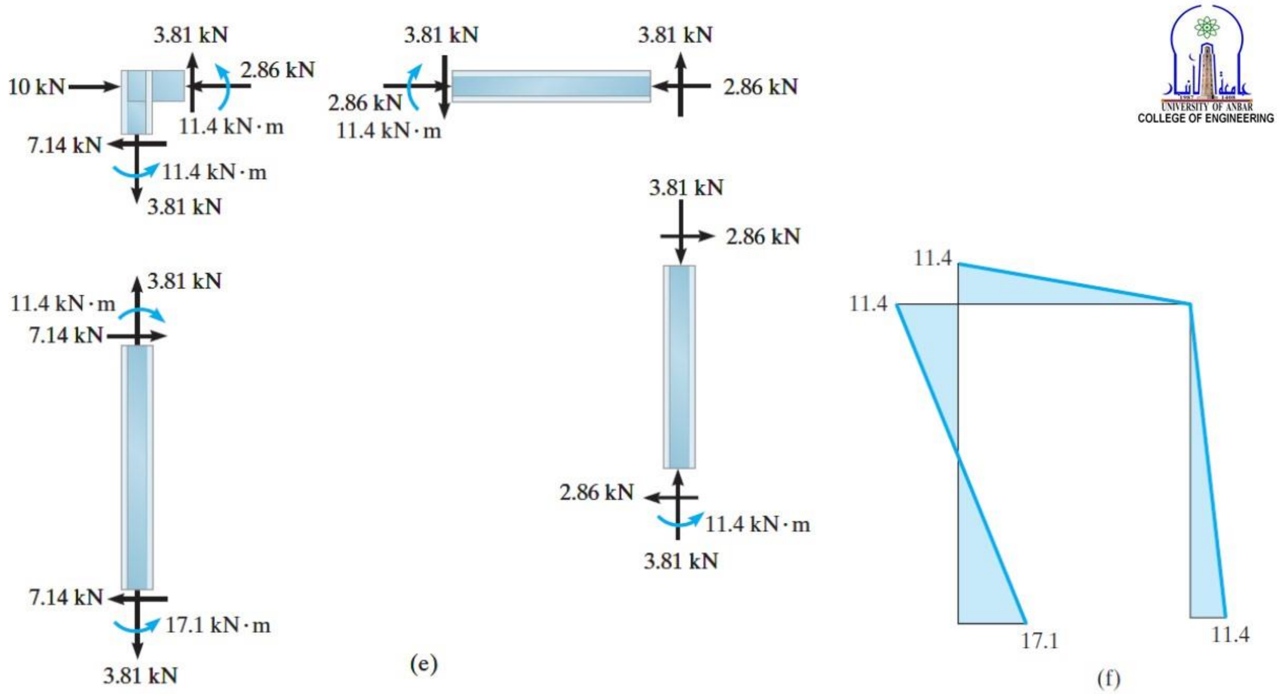

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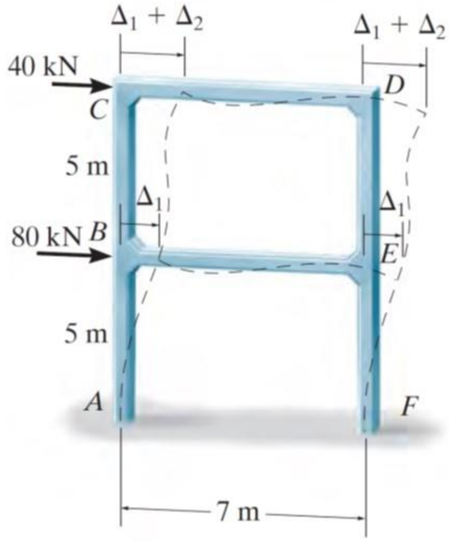
Example : Explain how the moments in each joint of the two-story frame shown in the figure are determined. EI is constant.

Solution :



$$M_{AB} = 2E\left(\frac{I}{5}\right)[2(0) + \theta_B - 3\psi_1] + 0$$

$$M_{BA} = 2E\left(\frac{I}{5}\right)[2\theta_B + 0 - 3\psi_1] + 0$$

$$M_{BC} = 2E\left(\frac{I}{5}\right)[2\theta_B + \theta_C - 3\psi_2] + 0$$

$$M_{CB} = 2E\left(\frac{I}{5}\right)[2\theta_C + \theta_B - 3\psi_2] + 0$$

$$M_{CD} = 2E\left(\frac{I}{7}\right)[2\theta_C + \theta_D - 3(0)] + 0$$

$$M_{DC} = 2E\left(\frac{I}{7}\right)[2\theta_D + \theta_C - 3(0)] + 0$$

$$M_{BE} = 2E\left(\frac{I}{7}\right)[2\theta_B + \theta_E - 3(0)] + 0$$

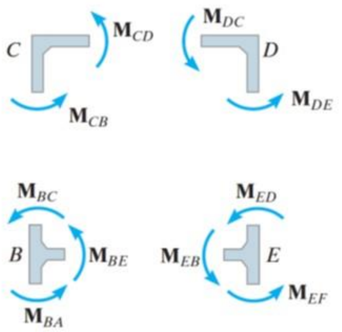
$$M_{EB} = 2E\left(\frac{I}{7}\right)[2\theta_E + \theta_B - 3(0)] + 0$$

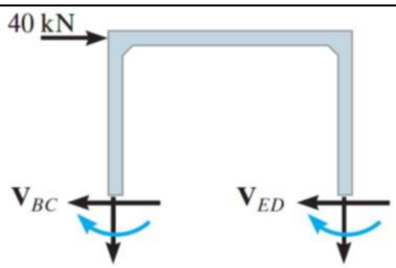
$$M_{ED} = 2E\left(\frac{I}{5}\right)[2\theta_E + \theta_D - 3\psi_2] + 0$$

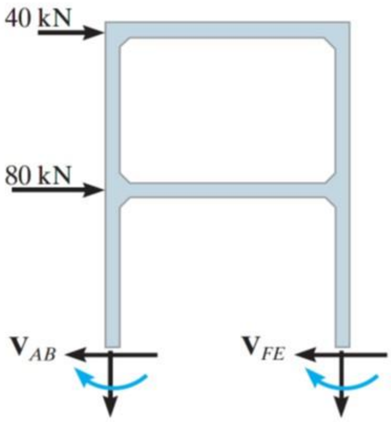
$$M_{DE} = 2E\left(\frac{I}{5}\right)[2\theta_D + \theta_E - 3\psi_2] + 0$$

$$M_{FE} = 2E\left(\frac{I}{5}\right)[2(0) + \theta_E - 3\psi_1] + 0$$

$$M_{EF} = 2E\left(\frac{I}{5}\right)[2\theta_E + 0 - 3\psi_1] + 0$$







$$M_{BA} + M_{BE} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

$$M_{DC} + M_{DE} = 0$$

$$M_{EF} + M_{EB} + M_{ED} = 0$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 40 - V_{BC} - V_{ED} = 0$$

$$40 + \frac{M_{BC} + M_{CB}}{5} + \frac{M_{ED} + M_{DE}}{5} = 0$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 40 + 80 - V_{AB} - V_{FE} = 0$$

$$120 + \frac{M_{AB} + M_{BA}}{5} + \frac{M_{EF} + M_{FE}}{5} = 0$$

Substituting the 12 slope-deflection equations in these 6 equilibrium equations will lead to a system of 6-equatins 6-unknowns which can be solve algebraically to find :

$$\psi_1, \psi_2, \theta_B, \theta_C, \theta_D, \text{ and } \theta_E$$

Then Moments can be found and drawn

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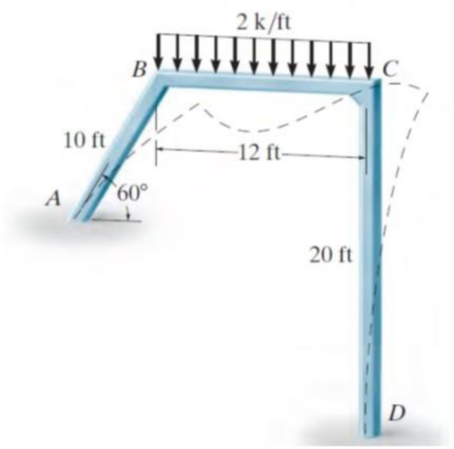
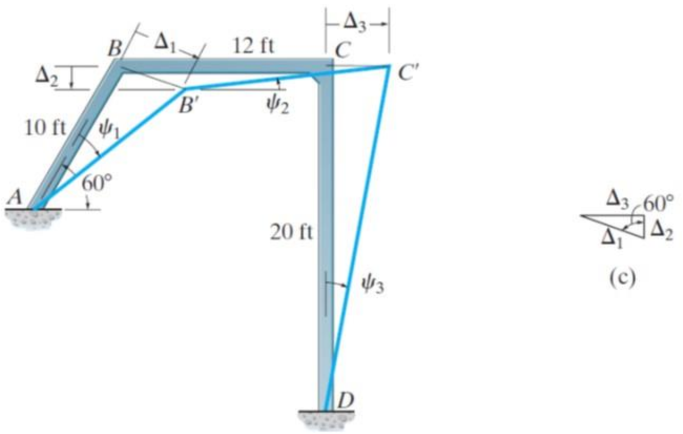
Example : Determine the moments at each joint of the frame shown in the figure. EI is constant for each member.

Solution :





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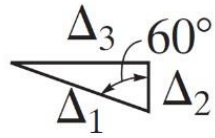
$$(\text{FEM})_{BC} = -\frac{wL^2}{12} = -\frac{2(12)^2}{12} = -24\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = \frac{wL^2}{12} = \frac{2(12)^2}{12} = 24\ \text{k}\cdot\text{ft}$$

$$\psi_1 = \frac{\Delta_1}{10} \qquad \psi_2 = -\frac{\Delta_2}{12} \qquad \psi_3 = \frac{\Delta_3}{20}$$



But: $\Delta_2 = 0.5\Delta_1$ and $\Delta_3 = 0.866\Delta_1$

$$\psi_2 = -0.417\psi_1 \qquad \psi_3 = 0.433\psi_1$$

$$M_{AB} = 2E\left(\frac{I}{10}\right)[2(0) + \theta_B - 3\psi_1] + 0 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{10}\right)[2\theta_B + 0 - 3\psi_1] + 0 \qquad (2)$$

$$M_{BC} = 2E\left(\frac{I}{12}\right)[2\theta_B + \theta_C - 3(-0.417\psi_1)] - 24 \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{12}\right)[2\theta_C + \theta_B - 3(-0.417\psi_1)] + 24 \qquad (4)$$

$$M_{CD} = 2E\left(\frac{I}{20}\right)[2\theta_C + 0 - 3(0.433\psi_1)] + 0 \qquad (5)$$

$$M_{DC} = 2E\left(\frac{I}{20}\right)[2(0) + \theta_C - 3(0.433\psi_1)] + 0 \qquad (6)$$

$$M_{BA} + M_{BC} = 0 \qquad (7)$$

$$M_{CD} + M_{CB} = 0 \qquad (8)$$

$$\curvearrowleft + \Sigma M_O = 0;$$

$$M_{AB} + M_{DC} - \left(\frac{M_{AB} + M_{BA}}{10}\right)(34) - \left(\frac{M_{DC} + M_{CD}}{20}\right)(40.78) - 24(6) = 0$$

$$-2.4M_{AB} - 3.4M_{BA} - 2.04M_{CD} - 1.04M_{DC} - 144 = 0 \qquad (9)$$

$$0.733\theta_B + 0.167\theta_C - 0.392\psi_1 = \frac{24}{EI}$$

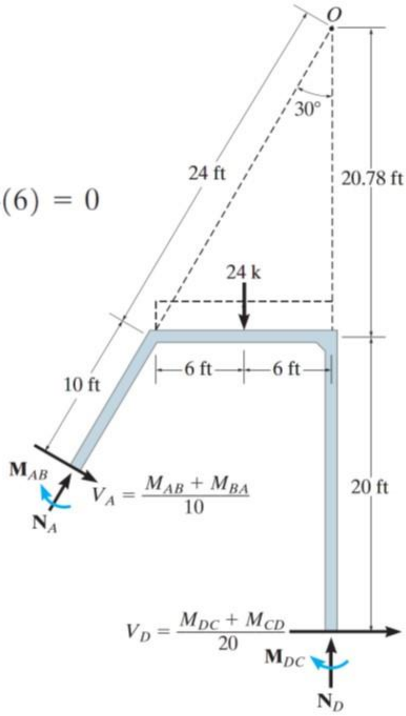
$$0.167\theta_B + 0.533\theta_C + 0.0784\psi_1 = -\frac{24}{EI}$$

$$-1.840\theta_B - 0.512\theta_C + 3.880\psi_1 = \frac{144}{EI}$$

$$EI\theta_B = 87.67 \qquad EI\theta_C = -82.3 \qquad EI\psi_1 = 67.83$$

$$M_{AB} = -23.2\ \text{k}\cdot\text{ft} \quad M_{BC} = 5.63\ \text{k}\cdot\text{ft} \quad M_{CD} = -25.3\ \text{k}\cdot\text{ft}$$

$$M_{BA} = -5.63\ \text{k}\cdot\text{ft} \quad M_{CB} = 25.3\ \text{k}\cdot\text{ft} \quad M_{DC} = -17.0\ \text{k}\cdot\text{ft}$$



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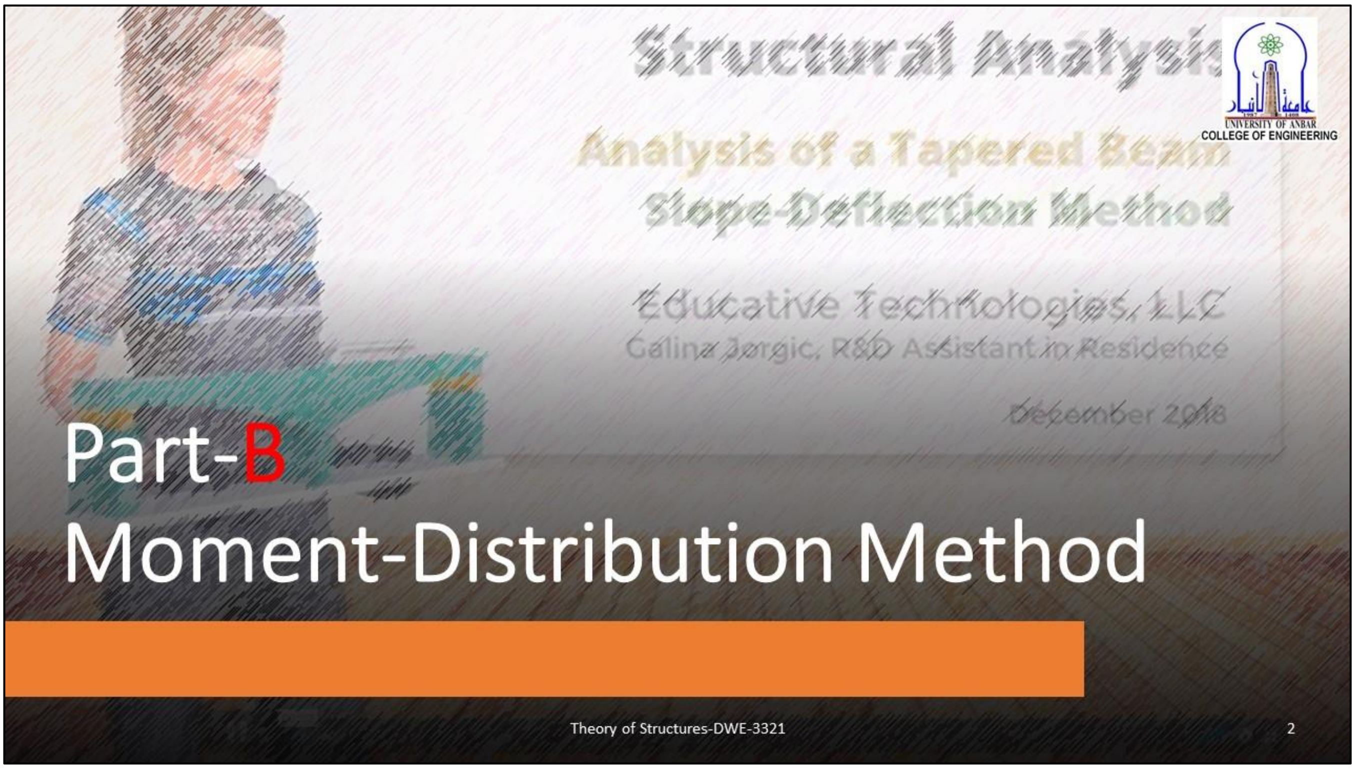
Unit-8

Analysis of Indeterminate Structures Using Displacement Methods

Theory of Structures-DWE-3321 1



Moment-Distribution Method

The method of analysing beams and frames using moment distribution was developed by Hardy Cross, in 1930. At the time this method was first published it attracted immediate attention, and it has been recognized as one of the most notable advances in structural analysis during the twentieth century.

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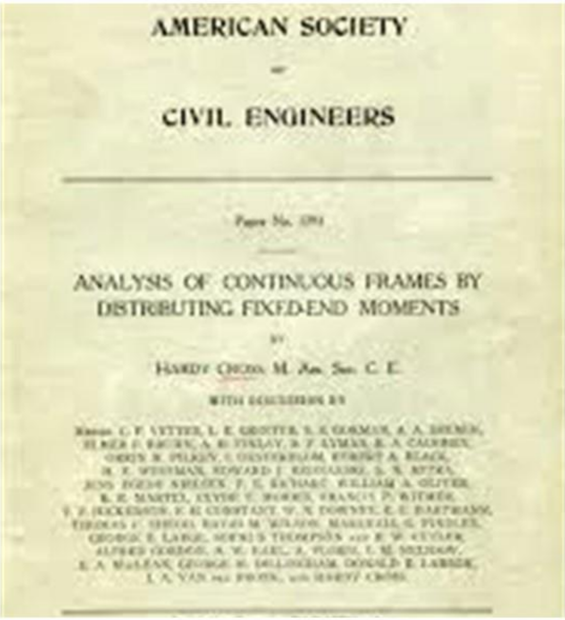
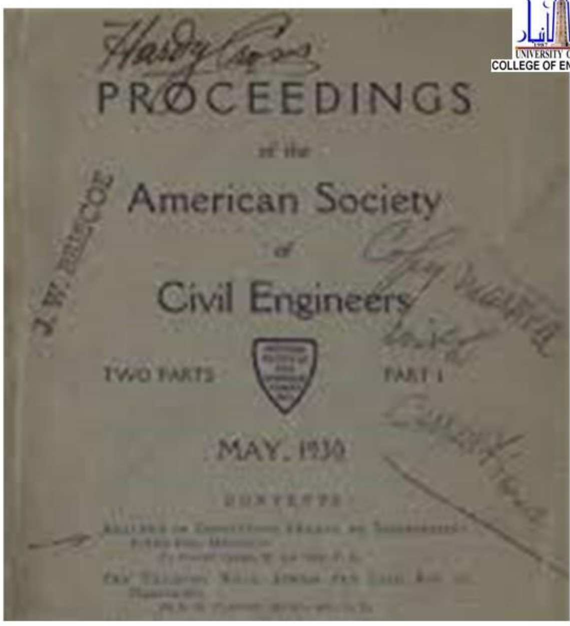

AMERICAN SOCIETY

CIVIL ENGINEERS

ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS

HARDY CROSS, M. AM. SOC. C. E.

WITH DISCUSSION BY


PROCEEDINGS

of the

American Society

of

Civil Engineers

TWO PARTS PART 1

MAY, 1930

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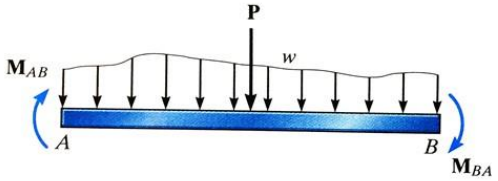
Sign Convention :



Clockwise moments that act on the member are considered ***positive***, whereas ***counterclockwise*** moments are ***negative***

Fixed-End Moments (FEMs) :

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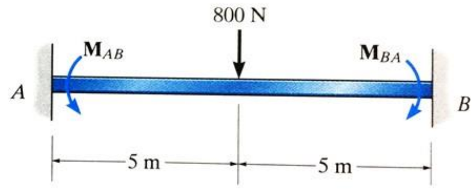
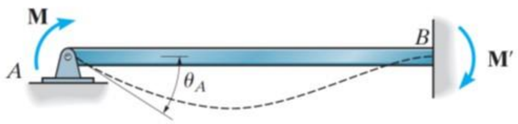


FEM = PL/8 = 800(10)/8 = 1000 N.m.

Noting the action of these moments on the beam and applying our sign convention, it is seen that

$M_{AB} = -1000$ N.m $\quad M_{BA} = +1000$ N.m

Member Stiffness Factor :



$$\boxed{M = (4EI/L)\,\theta_A}$$

$$K = \frac{4EI}{L}$$

Far End Fixed

K is referred to as the stiffness factor at ***A*** and can be defined as the amount of moment ***M*** required to rotate the end ***A*** of the beam $\theta_A = 1$ rad.

Joint Stiffness Factor :

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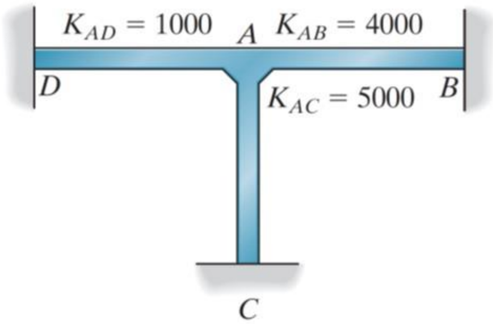
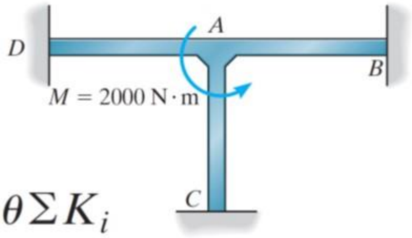


$$\boxed{K_T = \Sigma K}$$

$$K_T = \Sigma K = 4000 + 5000 + 1000$$
$$= 10\,000.$$

Distribution Factor (DF) :

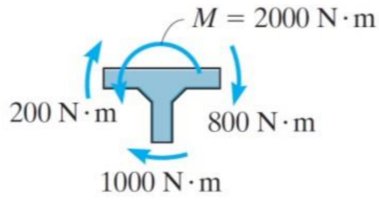


$$M_i = K_i\theta$$

$$M = M_1 + M_n = K_1\theta + K_n\theta = \theta\Sigma K_i$$

$$\boxed{\mathrm{DF}_i = \frac{M_i}{M} = \frac{K_i\theta}{\theta\Sigma K_i}}$$

$$\boxed{\mathrm{DF} = \frac{K}{\Sigma K}}$$



$\mathrm{DF}_{AB} = 4000/10\,000 = 0.4 \quad M_{AB} = 0.4(2000) = 800\ \mathrm{N\cdot m}$

$\mathrm{DF}_{AC} = 5000/10\,000 = 0.5 \quad M_{AC} = 0.5(2000) = 1000\ \mathrm{N\cdot m}$

$\mathrm{DF}_{AD} = 1000/10\,000 = 0.1 \quad M_{AD} = 0.1(2000) = 200\ \mathrm{N\cdot m}$

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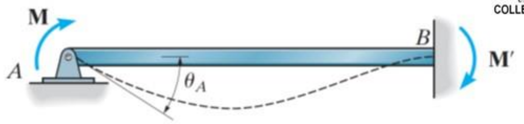
Member Relative Stiffness Factor :

Most of the time E is identical for all members, so it can be omitted from the equation :

$$\boxed{K_R = \frac{I}{L} \quad \text{Far End Fixed}}$$

Carry-Over Factor :



$$M_{AB} = (4EI/L)\,\theta_A$$

$$M_{BA} = (2EI/L)\,\theta_A$$

Solving for θ_A and equating the equations leads to the fact that :

$$M_{BA} = M_{AB}/2$$

$$\mathbf{M}' = \boxed{\frac{1}{2}}\mathbf{M}$$

COF

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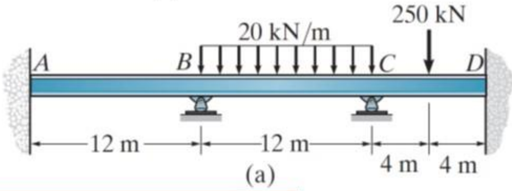
Example : Determine the internal moments at each support of the beam shown in the figure. EI is constant.

Solution :



$$K_{AB} = \frac{4EI}{12} \qquad K_{BC} = \frac{4EI}{12} \qquad K_{CD} = \frac{4EI}{8}$$

Therefore,

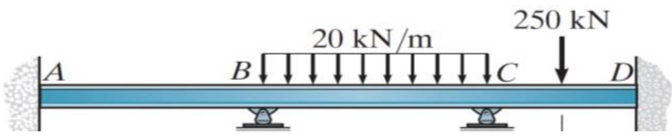
$$DF_{AB} = DF_{DC} = 0 \qquad DF_{BA} = DF_{BC} = \frac{4EI/12}{4EI/12 + 4EI/12} = 0.5$$

$$DF_{CB} = \frac{4EI/12}{4EI/12 + 4EI/8} = 0.4 \quad DF_{CD} = \frac{4EI/8}{4EI/12 + 4EI/8} = 0.6$$

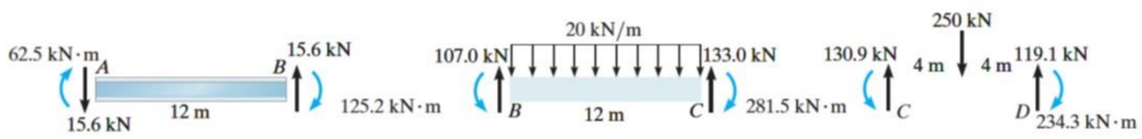
The fixed-end moments are

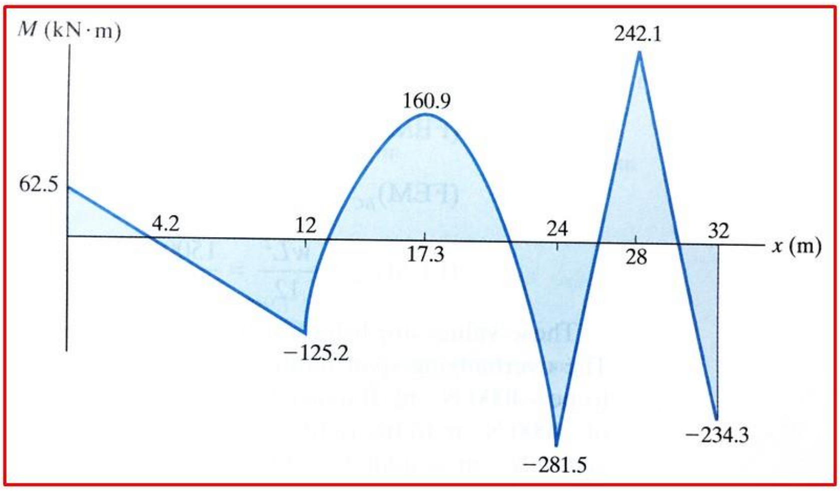
$$(FEM)_{BC} = -\frac{wL^2}{12} = \frac{-20(12)^2}{12} = -240\ \text{kN}\cdot\text{m} \qquad (FEM)_{CB} = \frac{wL^2}{12} = \frac{20(12)^2}{12} = 240\ \text{kN}\cdot\text{m}$$

$$(FEM)_{CD} = -\frac{PL}{8} = \frac{-250(8)}{8} = -250\ \text{kN}\cdot\text{m} \qquad (FEM)_{DC} = \frac{PL}{8} = \frac{250(8)}{8} = 250\ \text{kN}\cdot\text{m}$$



Joint	*A*	*B*		*C*		*D*	1
Member	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*	2
DF	0	0.5	0.5	0.4	0.6	0	3
FEM			−240	240	−250	250	4
Dist.		120	120	4	6		5
CO	60		2	60		3	6
Dist.		−1	−1	−24	−36		7
CO	−0.5		−12	−0.5		−18	8
Dist.		6	6	0.2	0.3		9
CO	3		0.1	3		0.2	10
Dist.		−0.05	−0.05	−1.2	−1.8		11
CO	−0.02		−0.6	−0.02		−0.9	12
Dist.		0.3	0.3	0.01	0.01		13
ΣM	62.5	125.2	−125.2	281.5	−281.5	234.3	14





Example : Determine the internal moments at each support of the beam shown in the figure. EI is constant and The moment of inertia of each span is indicated

Solution :

$$K_{BC} = \frac{4E(750)}{20} = 150E \qquad K_{CD} = \frac{4E(600)}{15} = 160E$$

$$\text{DF}_{BC} = 1 - (\text{DF})_{BA} = 1 - 0 = 1$$

$$\text{DF}_{CB} = \frac{150E}{150E + 160E} = 0.484$$

$$\text{DF}_{CD} = \frac{160E}{150E + 160E} = 0.516$$

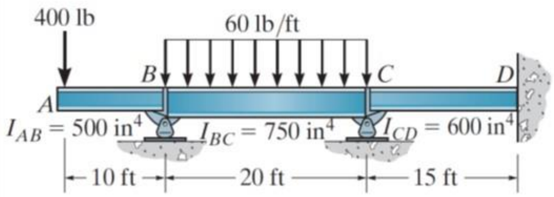
$$\text{DF}_{DC} = \frac{160E}{\infty + 160E} = 0$$

Due to the overhang,

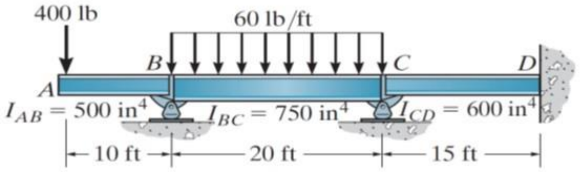
$$(\text{FEM})_{BA} = 400\text{ lb}(10\text{ ft}) = 4000\text{ lb}\cdot\text{ft}$$

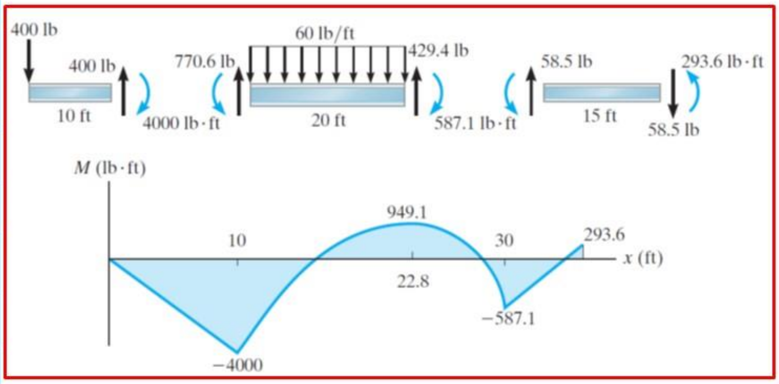
$$(\text{FEM})_{BC} = -\frac{wL^2}{12} = -\frac{60(20)^2}{12} = -2000\text{ lb}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = \frac{wL^2}{12} = \frac{60(20)^2}{12} = 2000\text{ lb}\cdot\text{ft}$$



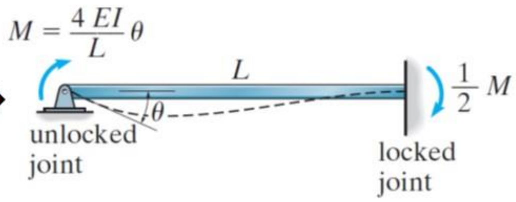
Joint	B		C		D
Member		BC	CB	CD	DC
DF	0	1	0.484	0.516	0
FEM	4000	−2000	2000		
Dist.		−2000	−968	−1032	
CO		−484	−1000		−516
Dist.		484	484	516	
CO		242	242		258
Dist.		−242	−117.1	−124.9	
CO		−58.6	−121		−62.4
Dist.		58.6	58.6	62.4	
CO		29.3	29.3		31.2
Dist.		−29.3	−14.2	−15.1	
CO		−7.1	−14.6		−7.6
Dist.		7.1	7.1	7.6	
CO		3.5	3.5		3.8
Dist.		−3.5	−1.7	−1.8	
CO		−0.8	−1.8		−0.9
Dist.		0.8	0.9	0.9	
CO		0.4	0.4		0.4
Dist.		−0.4	−0.2	−0.2	
CO		−0.1	−0.2		−0.1
Dist.		0.1	0.1	0.1	
ΣM	4000	−4000	587.1	−587.1	−293.6





Stiffness Factor Modifications :

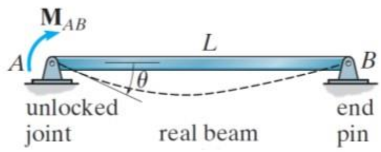
Typical Scenario !



Member Pin Supported at Far End :

$$\circlearrowleft + \Sigma M_{B'} = 0; \qquad V'_A(L) - \frac{1}{2}\left(\frac{M}{EI}\right)L\left(\frac{2}{3}L\right) = 0$$

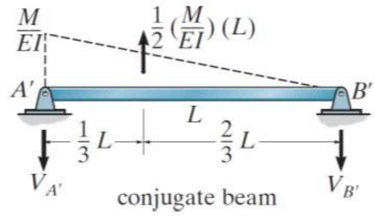
$$V'_A = \theta = \frac{ML}{3EI} \rightarrow \boxed{M = \frac{3EI}{L}\theta}$$



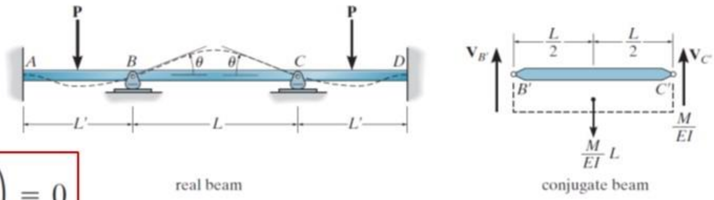
$$K = \frac{3EI}{L}$$

Far End Pinned or Roller Supported

∴ k would have to be modified by ¾ to model the case of having the far end pin connected.



Symmetric Beam and Loading :



$$\circlearrowleft + \Sigma M_{C'} = 0; \qquad -V_{B'}(L) + \frac{M}{EI}(L)\left(\frac{L}{2}\right) = 0$$

$$V_{B'} = \theta = \frac{ML}{2EI}$$

or

$$M = \frac{2EI}{L}\theta$$

$$K = \frac{2EI}{L}$$

Symmetric Beam and Loading

∴ *Thus, moments for only half the beam can be distributed provided the stiffness factor for the centre span is computed using* **K = 2EI/L** . *By comparison, the centre span's stiffness factor will be one* ***1/2*** *that usually determined using* **K = 4EI/L** .

Symmetric Beam with Antisymmetric Loading :

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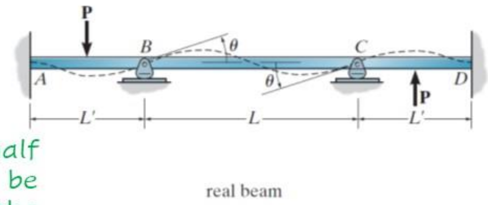
$$\circlearrowleft + \Sigma M_{C'} = 0; \; -V_{B'}(L) + \frac{1}{2}\left(\frac{M}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{5L}{6}\right) - \frac{1}{2}\left(\frac{M}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) = 0$$

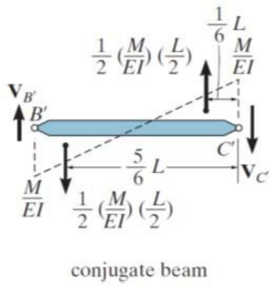
$$V_{B'} = \theta = \frac{ML}{6EI} \Rightarrow \boxed{M = \frac{6EI}{L}\theta}$$



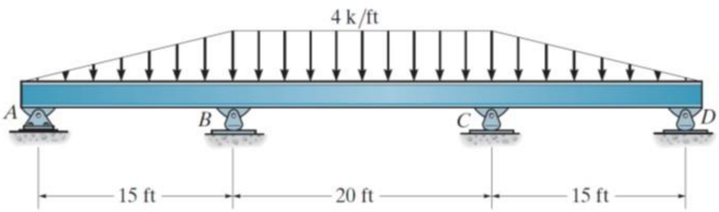
$$K = \frac{6EI}{L}$$

Symmetric Beam with Antisymmetric Loading

∴ *Thus, moments for only half the beam can be distributed provided the stiffness factor for the centre span is computed using* **K = 6EI/L** . *By comparison, the centre span's stiffness factor will be one* ***1.5*** *that usually determined using* **K = 4EI/L**.



Example : Determine the internal moments at each support of the beam shown in the figure. *EI* is constant.



Solution :

$$K_{AB} = \frac{3EI}{15}$$

$$K_{BC} = \frac{2EI}{20}$$

$$\text{DF}_{AB} = \frac{3EI/15}{3EI/15} = 1$$

$$\text{DF}_{BA} = \frac{3EI/15}{3EI/15 + 2EI/20} = 0.667$$

$$\text{DF}_{BC} = \frac{2EI/20}{3EI/15 + 2EI/20} = 0.333$$

$$(\text{FEM})_{BA} = \frac{wL^2}{15} = \frac{4(15)^2}{15} = 60 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = -\frac{wL^2}{12} = -\frac{4(20)^2}{12} = -133.3 \text{ k}\cdot\text{ft}$$

Joint	*A*	*B*	
Member	*AB*	*BA*	*BC*
DF	1	0.667	0.333
FEM Dist.		60 48.9	−133.3 24.4
ΣM	0	108.9	−108.9

Example : Determine the internal moments at each support of the beam shown in the figure. *The moments of inertia for the two spans are indicated.*

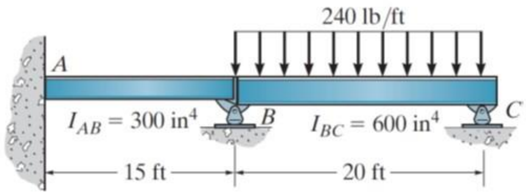
Solution :

$$K_{AB} = \frac{4EI}{L} = \frac{4E(300)}{15} = 80E \qquad \text{DF}_{AB} = \frac{80E}{\infty + 80E} = 0$$

$$K_{BC} = \frac{3EI}{L} = \frac{3E(600)}{20} = 90E \qquad \text{DF}_{BA} = \frac{80E}{80E + 90E} = 0.4706$$

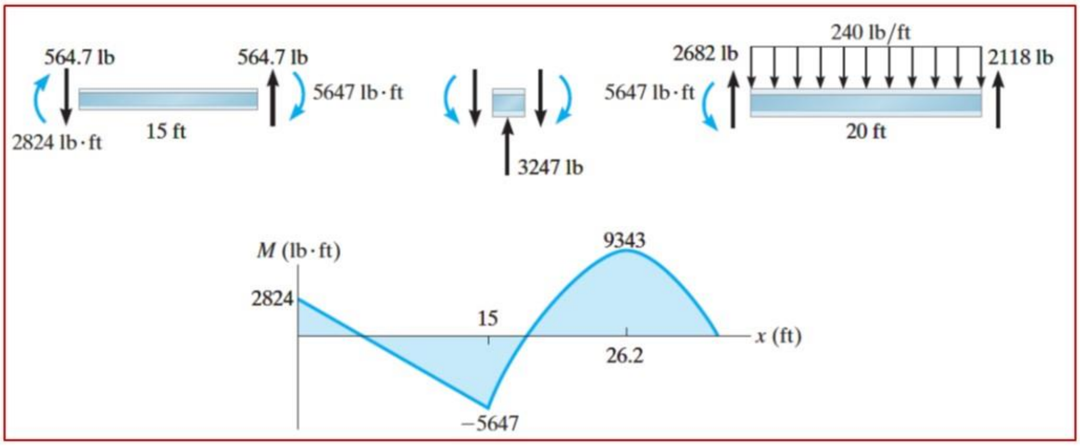
$$\text{DF}_{BC} = \frac{90E}{80E + 90E} = 0.5294$$

$$\text{DF}_{CB} = \frac{90E}{90E} = 1$$



$$(\text{FEM})_{BC} = -\frac{wL^2}{8} = \frac{-240(20)^2}{8} = -12\,000 \text{ lb}\cdot\text{ft}$$

Joint	*A*	*B*		*C*
Member	*AB*	*BA*	*BC*	*CB*
DF	0	0.4706	0.5294	1
FEM Dist.		 5647.2	−12 000 6352.8	
CO	2823.6			
ΣM	2823.6	5647.2	−5647.2	0



Moment Distribution for Frames: NO SIDESWAY

Example : Determine the internal moments at the joints of the frame shown in the figure. *There is a pin at **E** and **D** and a fixed support at **A**. EI is constant.*

Solution :

$$K_{AB} = \frac{4EI}{15} \qquad K_{BC} = \frac{4EI}{18} \qquad K_{CD} = \frac{3EI}{15} \qquad K_{CE} = \frac{3EI}{12}$$

$$\text{DF}_{AB} = 0$$

$$\text{DF}_{BA} = \frac{4EI/15}{4EI/15 + 4EI/18} = 0.545$$

$$\text{DF}_{BC} = 1 - 0.545 = 0.455$$

$$\text{DF}_{CB} = \frac{4EI/18}{4EI/18 + 3EI/15 + 3EI/12} = 0.330$$

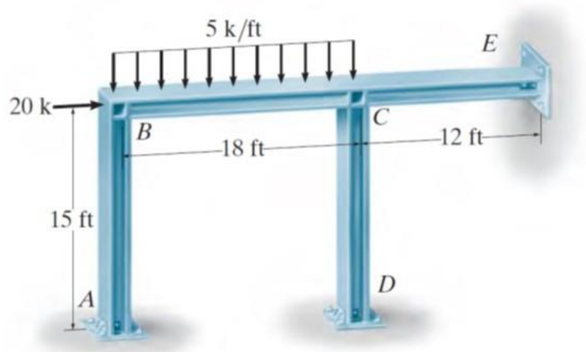
$$\text{DF}_{CD} = \frac{3EI/15}{4EI/18 + 3EI/15 + 3EI/12} = 0.298$$

$$\text{DF}_{CE} = 1 - 0.330 - 0.298 = 0.372$$

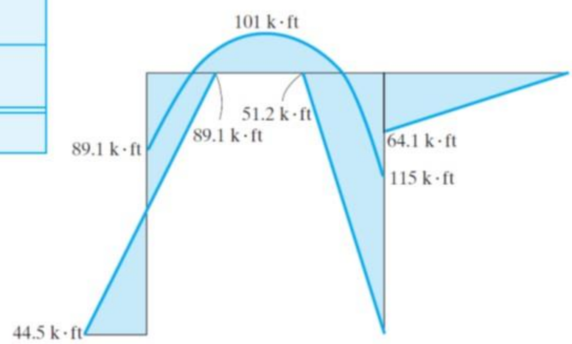
$$\text{DF}_{DC} = 1 \qquad \text{DF}_{EC} = 1$$

$$(\text{FEM})_{BC} = \frac{-wL^2}{12} = \frac{-5(18)^2}{12} = -135 \text{ k} \cdot \text{ft}$$

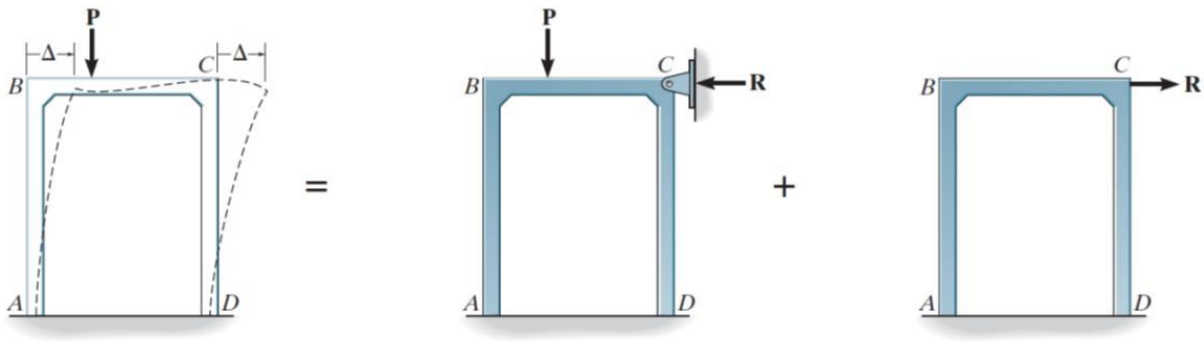
$$(\text{FEM})_{CB} = \frac{wL^2}{12} = \frac{5(18)^2}{12} = 135 \text{ k} \cdot \text{ft}$$



Joint	A	B		C			D	E
Member	AB	BA	BC	CB	CD	CE	DC	EC
DF	0	0.545	0.455	0.330	0.298	0.372	1	1
FEM			−135	135				
Dist.		73.6	61.4	−44.6	−40.2	−50.2		
CO	36.8		−22.3	30.7				
Dist.		12.2	10.1	−10.1	−9.1	−11.5		
CO	6.1		−5.1	5.1				
Dist.		2.8	2.3	−1.7	−1.5	−1.9		
CO	1.4		−0.8	1.2				
Dist.		0.4	0.4	−0.4	−0.4	−0.4		
CO	0.2		−0.2	0.2				
Dist.		0.1	0.1	−0.1	0.0	−0.1		
ΣM	44.5	89.1	−89.1	115	−51.2	−64.1		



Moment Distribution for Frames: SIDESWAY



artificial joint applied
(no sidesway)

artificial joint removed
(sidesway)

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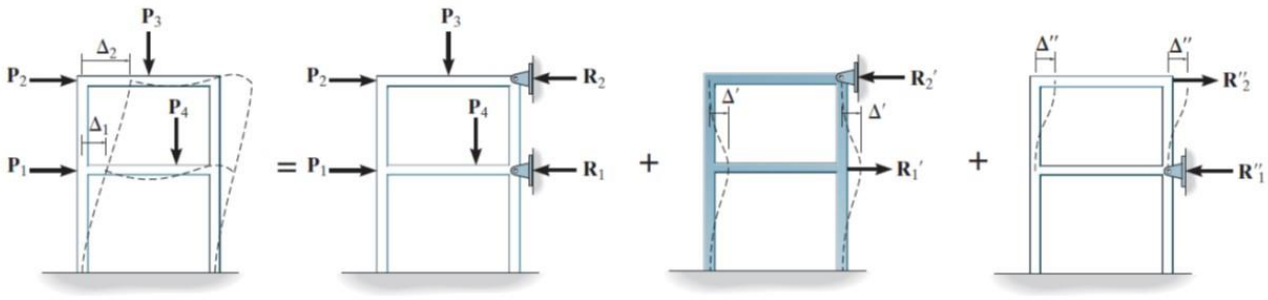

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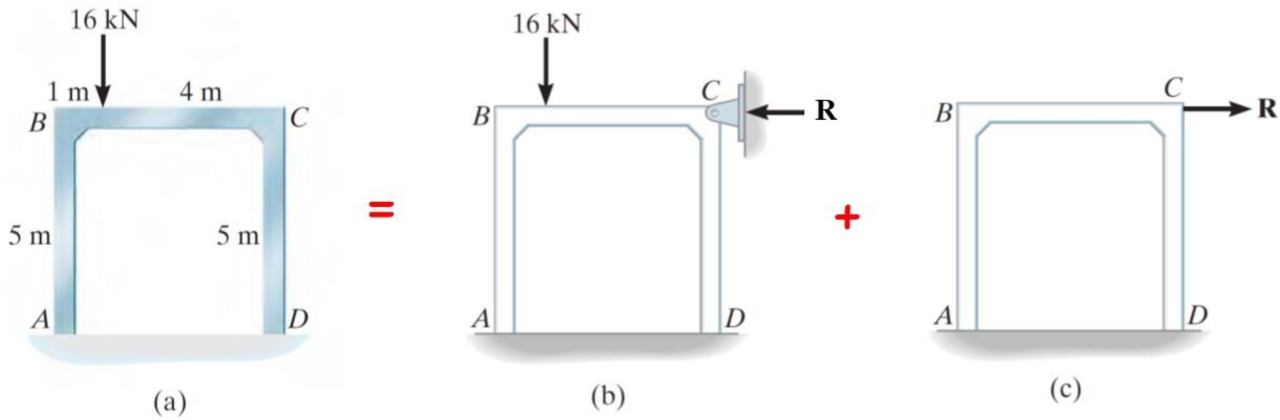
Example : Determine the moment at the joints of the frame shown in the figure. *EI is constant.*

No-Sway Solution :

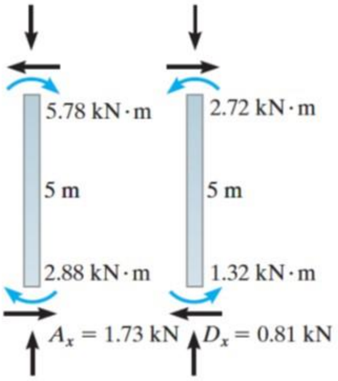


$$(\text{FEM})_{BC} = -\frac{16(4)^2(1)}{(5)^2} = -10.24 \text{ kN}\cdot\text{m}$$

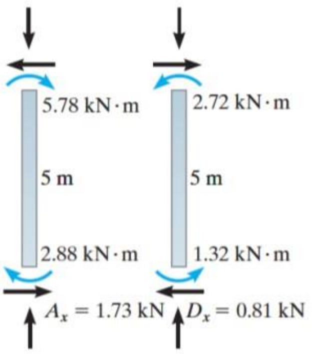
$$(\text{FEM})_{CB} = \frac{16(1)^2(4)}{(5)^2} = 2.56 \text{ kN}\cdot\text{m}$$

Joint	*A*	*B*		*C*		*D*
Member	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*
DF	0	0.5	0.5	0.5	0.5	0
FEM			−10.24	2.56		
Dist.		5.12	5.12	−1.28	−1.28	
CO	2.56		−0.64	2.56		−0.64
Dist.		0.32	0.32	−1.28	−1.28	
CO	0.16		−0.64	0.16		−0.64
Dist.		0.32	0.32	−0.08	−0.08	
CO	0.16		−0.04	0.16		−0.04
Dist.		0.02	0.02	−0.08	−0.08	
ΣM	2.88	5.78	−5.78	2.72	−2.72	−1.32

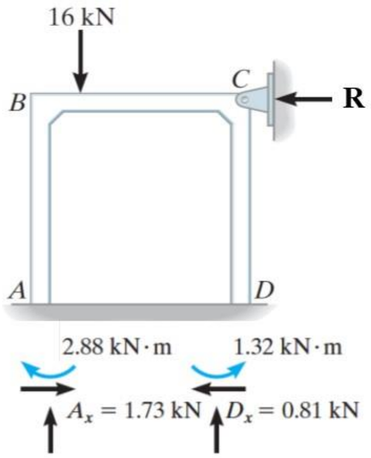
(d)



(e)



⇒



$$A_x = \frac{5.78 + 2.88}{5} = 1.73\ kN$$

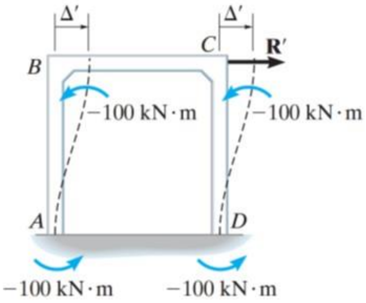
$$D_x = \frac{2.72 + 1.32}{5} = 0.81\ kN$$

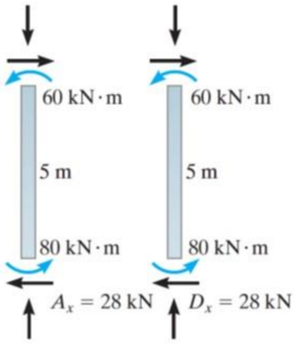
$$\boxed{\Sigma F_x = 0; \qquad R = 1.73 \text{ kN} - 0.81 \text{ kN} = 0.92 \text{ kN}}$$

Sway Solution : We will arbitrarily assume the FEM to be 100 kN.m

$$M = \frac{6EI\Delta}{L^2} \Rightarrow (\text{FEM})_{AB} = (\text{FEM})_{BA} = (\text{FEM})_{CD} = (\text{FEM})_{DC} = -100\ \text{kN}\cdot\text{m}$$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0	0.5	0.5	0.5	0.5	0
FEM	−100	−100			−100	−100
Dist.		50	50	50	50	
CO	25		25	25		25
Dist.		−12.5	−12.5	−12.5	−12.5	
CO	−6.25		−6.25	−6.25		−6.25
Dist.		3.125	3.125	3.125	3.125	
CO	1.56		1.56	1.56		1.56
Dist.		−0.78	−0.78	−0.78	−0.78	
CO	−0.39		−0.39	−0.39		−0.39
Dist.		0.195	0.195	0.195	0.195	
ΣM	−80.00	−60.00	60.00	60.00	−60.00	−80.00





$\Sigma F_x = 0;\qquad R' = 28 + 28 = 56.0\ \text{kN}$

Total / Final Solution = NoSway + Modified Sway Solutions

$$M_{AB} = 2.88 + \tfrac{0.92}{56.0}(-80) = 1.57\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

$$M_{BA} = 5.78 + \tfrac{0.92}{56.0}(-60) = 4.79\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

$$M_{BC} = -5.78 + \tfrac{0.92}{56.0}(60) = -4.79\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

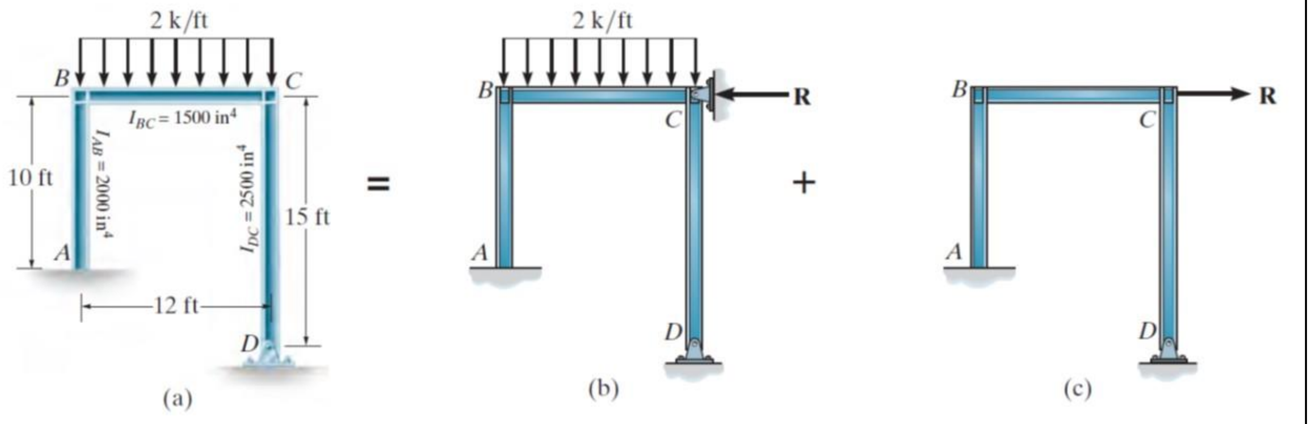
$$M_{CB} = 2.72 + \tfrac{0.92}{56.0}(60) = 3.71\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

$$M_{CD} = -2.72 + \tfrac{0.92}{56.0}(-60) = -3.71\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

$$M_{DC} = -1.32 + \tfrac{0.92}{56.0}(-80) = -2.63\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

Example : Determine the moment at the joints of the frame shown in the figure. *The moment of inertia is indicated.*

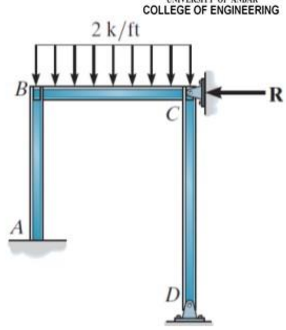
Solution :



No-Sway Solution :

$$FEM_{BC} = -\frac{w\,l^2}{12} = -\frac{2 \times 12^2}{12} = -24\ k.ft \qquad FEM_{CB} = \frac{w\,l^2}{12} = \frac{2 \times 12^2}{12} = 24\ k.ft$$

$$K_{AB} = \frac{4E(2000)}{10} = 800\ E \qquad K_{BC} = \frac{4E(1500)}{12} = 500\ E \qquad K_{CD} = \frac{3E(2500)}{15} = 500\ E$$



$DF_{AB} = 0$

$$DF_{BA} = \frac{800E}{800E + 500E} = 0.615$$

$$DF_{BC} = \frac{500E}{800E + 500E} = 0.385$$

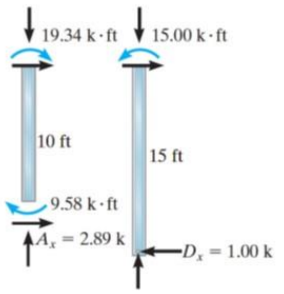
$$DF_{CB} = \frac{500E}{500E + 500E} = 0.5$$

$$DF_{CB} = \frac{500E}{500E + 500E} = 0.5$$

$DF_{DC} = 1$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0	0.615	0.385	0.5	0.5	1
FEM			−24	24		
Dist.		14.76	9.24	−12	−12	
CO	7.38		−6	4.62		
Dist.		3.69	2.31	−2.31	−2.31	
CO	1.84		−1.16	1.16		
Dist.		0.713	0.447	−0.58	−0.58	
CO	0.357		−0.29	0.224		
Dist.		0.18	0.11	−0.11	−0.11	
ΣM	9.58	19.34	−19.34	15.00	−15.00	0

$$\boxed{\Sigma F_x = 0; \qquad R = 2.89 - 1.00 = 1.89\ k}$$

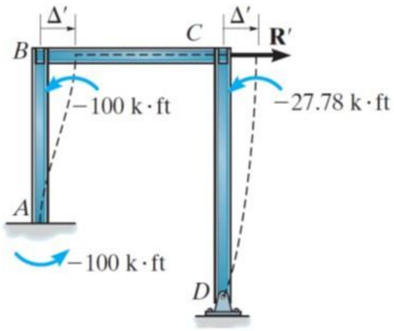


Sway Solution :

$$(\mathrm{FEM})_{AB} = (\mathrm{FEM})_{BA} = -\frac{6EI\Delta}{L^2} = -\frac{6E(2000)\Delta'}{(10)^2}$$

$$(\mathrm{FEM})_{CD} = -\frac{3EI\Delta}{L^2} = -\frac{3E(2500)\Delta'}{(15)^2}$$

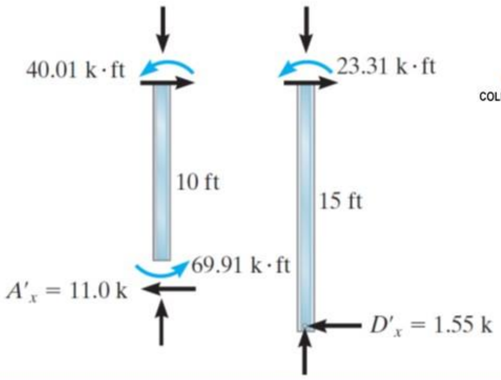
Assuming the FEM for AB is -100 k.ft, the corresponding FEM at C, causing the same Δ' is found by comparison, i.e.,



$$\Delta' = -\frac{(-100)(10)^2}{6E(2000)} = -\frac{(\mathrm{FEM})_{CD}(15)^2}{3E(2500)}$$

$$(\mathrm{FEM})_{CD} = -27.78\ \mathrm{k \cdot ft}$$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0	0.615	0.385	0.5	0.5	1
FEM	−100	−100			−27.78	
Dist.		61.5	38.5	13.89	13.89	
CO	30.75		6.94	19.25		
Dist.		−4.27	−2.67	−9.625	−9.625	
CO	−2.14		−4.81	−1.34		
Dist.		2.96	1.85	0.67	0.67	
CO	1.48		0.33	0.92		
Dist.		−0.20	−0.13	−0.46	−0.46	
ΣM	−69.91	−40.01	40.01	23.31	−23.31	0



$$\Sigma F_x = 0; \qquad R' = 11.0 + 1.55 = 12.55\ \mathrm{k}$$

$$M_{AB} = 9.58 + \left(\tfrac{1.89}{12.55}\right)(-69.91) = -0.948\ \mathrm{k \cdot ft}$$

$$M_{BA} = 19.34 + \left(\tfrac{1.89}{12.55}\right)(-40.01) = 13.3\ \mathrm{k \cdot ft}$$

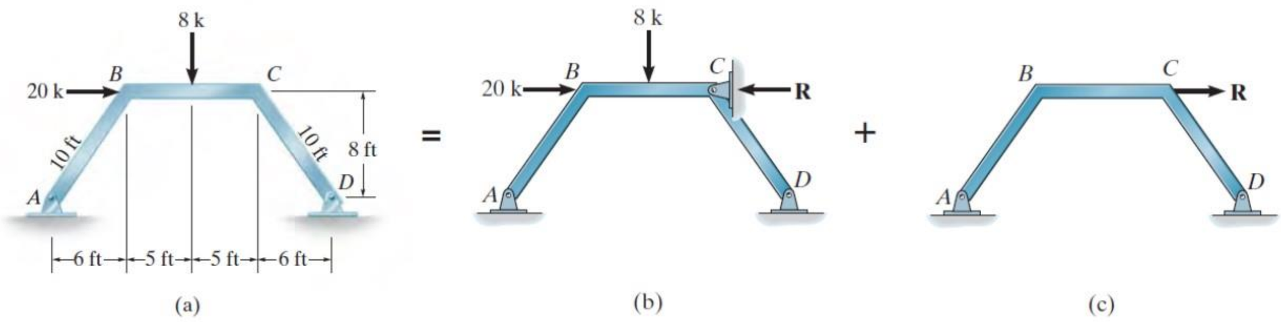
$$M_{BC} = -19.34 + \left(\tfrac{1.89}{12.55}\right)(40.01) = -13.3\ \mathrm{k \cdot ft}$$

$$M_{CB} = 15.00 + \left(\tfrac{1.89}{12.55}\right)(23.31) = 18.5\ \mathrm{k \cdot ft}$$

$$M_{CD} = -15.00 + \left(\tfrac{1.89}{12.55}\right)(-23.31) = -18.5\ \mathrm{k \cdot ft}$$

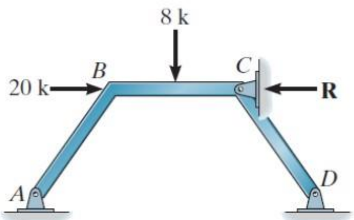
Example : Determine the moment at the joints of the frame shown in the figure. *EI is constant.*

Solution :

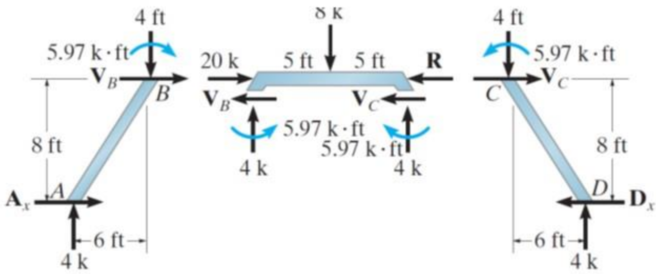


No-Sway Solution :

$$(\text{FEM})_{BC} = -\frac{8(10)}{8} = -10\ \text{k}\cdot\text{ft} \qquad (\text{FEM})_{CB} = \frac{8(10)}{8} = 10\ \text{k}\cdot\text{ft}$$



Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	1	0.429	0.571	0.571	0.429	1
FEM Dist.		4.29	−10 5.71	10 −5.71	−4.29	
CO Dist.		1.23	−2.86 1.63	2.86 −1.63	−1.23	
CO Dist.		0.35	−0.82 0.47	0.82 −0.47	−0.35	
CO Dist.		0.10	−0.24 0.13	0.24 −0.13	−0.10	
ΣM	0	5.97	−5.97	5.97	−5.97	0



$$\circlearrowleft + \Sigma M_B = 0; \qquad -5.97 + A_x(8) - 4(6) = 0 \qquad A_x = 3.75\ \text{k}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad 5.97 - D_x(8) + 4(6) = 0 \qquad D_x = 3.75\ \text{k}$$

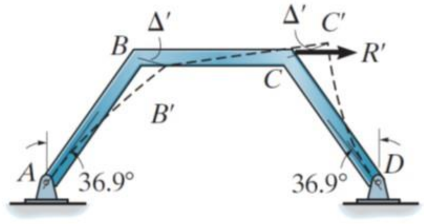
Thus, for the entire frame,

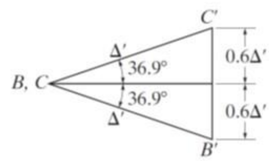
$$\Sigma F_x = 0; \qquad R = 3.75 - 3.75 + 20 = 20\ \text{k}$$

Sway Solution :

$(FEM)_{BA} = (FEM)_{CD} = -3EI\Delta'/(10)^2$, $(FEM)_{BC} = (FEM)_{CB} = 6EI(1.2\Delta')/(10)^2$.

If we arbitrarily assign a value of $(FEM)_{BA} = (FEM)_{CD} = -100\ \text{k}\cdot\text{ft}$, then equating Δ' in the above formulas yields $(FEM)_{BC} = (FEM)_{CB} = 240\ \text{k}\cdot\text{ft}$.





Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	1	0.429	0.571	0.571	0.429	1
FEM		−100	240	240	−100	
Dist.		−60.06	−79.94	−79.94	−60.06	
CO			−39.97	−39.97		
Dist.		17.15	22.82	22.82	17.15	
CO			11.41	11.41		
Dist.		−4.89	−6.52	−6.52	−4.89	
CO			−3.26	−3.26		
Dist.		1.40	1.86	1.86	1.40	
CO			0.93	0.93		
Dist.		−0.40	−0.53	−0.53	−0.40	
ΣM	0	−146.80	146.80	146.80	−146.80	0

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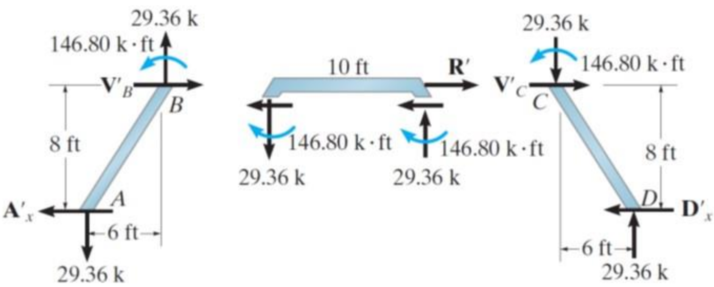

$\circlearrowleft + \Sigma M_B = 0; \quad -A'_x(8) + 29.36(6) + 146.80 = 0 \qquad A'_x = 40.37\ \text{k}$

$\circlearrowleft + \Sigma M_C = 0; \quad -D'_x(8) + 29.36(6) + 146.80 = 0 \qquad D'_x = 40.37\ \text{k}$

Thus, for the entire frame,

$\Sigma F_x = 0; \qquad R' = 40.37 + 40.37 = 80.74\ \text{k}$

$$M_{BA} = 5.97 + \left(\frac{20}{80.74}\right)(-146.80) = -30.4\ \text{k}\cdot\text{ft}$$

$$M_{BC} = -5.97 + \left(\frac{20}{80.74}\right)(146.80) = 30.4\ \text{k}\cdot\text{ft}$$

$$M_{CB} = 5.97 + \left(\frac{20}{80.74}\right)(146.80) = 42.3\ \text{k}\cdot\text{ft}$$

$$M_{CD} = -5.97 + \left(\frac{20}{80.74}\right)(-146.80) = -42.3\ \text{k}\cdot\text{ft}$$

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